I Strength of Material

• Basic of SOM

Rigid Body

The body which does not deform after application of external force is called rigid body.

Deformable Body

The body which deform when external force is applied on the body is called deformable body.

There are two types of deformable body.

1. Elastic Body
2. Plastic Body

Elastic Body

When an external force act on a body, the body tend to undergo some deformation.

If the external force is removed and body comes back to its original shape and size. The body is called elastic body.

Plastic Body

The body which do not regain it original shape and size after removal of external force or deforming force is called plastic body.

Elasticity

The property by vertue of which certain material return back to its original position after removal of external force is called Elasticity.

Plasticity

The property by vertue of which body does not regain its original shape and size after removal of forming force or external force called plasticity.

Deforming force

If a force is applied on a body its change the shape a[illegible]
Size of the body by producing a change in normal position
of its molecule such type of foce called deforming force.

Restoring force

The force which restrore the shape and size of body is called
restoring force or internal resisting force.

Mechanics

The branch of physics which deals with study of force
and the effect of forces on body is called mechanics.

Types of Mechanics

1. Engg. Mechanics
2. Mechanics of solid
3. Mechanics of fluid

Engineering Mechanics

The branch of mechanics which deals with the study of
force and their effect on rigid body is called Engineering
Mechanics.

Mechanics of solid

The branch of mechanics which deals with the study
of forces and their effect on deformable body is
called mechanics of solid.

Mechanics of fluid

The branch of mechanics which deals with the study
of forces on fluid at rest or motion is called mechanics
of fluid.

Strength of Material;

Strength of material mainly deal with the behaviour of Material when some external load is applied to them.

It is also known by mechanics of solid or mechanics of material.

According to material science, strength of material is defined as the ability of material to withstand and applied load without failure.

Aim of SOM

The aim of SOM is to derive the expression for deformation stress and strain developed in a component under different loading condition by using experimentally obtained elastic property like young modulus and poison's ratio.

Assumption made in strength of material,

- Material is assumed to be homogeneous and isotropic.
- Material obey Hook's Law.
- Component is assumed to be prismative (uniform cross-sectional dimension through out the length of member).
- Load is assume to be static load (Magnitude and direction through out the section).
- Self weight of component is neglected.

Homogeneous Material

A material is said to be homogeneous when it exhibit same elastic property at any point

Isotropic Material

A material is said to be isotropic when it exhibit same elastic properties at any point.

Homogeneous and Isotropic Material

A material is said to be both homogeneous and isotropic when it exihibit same elastic properties at any point and in any direction.

UNIT - 1 Simple stress & strain

Load

Load is define as the external force or couple to which component is subjected during its functionality.

It is denoted by P or M or T.

P → External Load → Newton

M → Bending Moment → Newton meter

T → twisting Moment → Newton meter

Classification of Load:

- According to cross-section:

1 Normal Load - A load which act perpendicular to cross section of a material is called normal load.

1. Axial Normal Load
2. Eccentric axial normal load

1. Axial Normal Load :

The load which act perpendicular to the cross-section and passes through the axis is axial normal Load.

2. Excentric Axial Normal Load:

The load which act perpendicular to the cross-section and passes away from the axis is Eccentric axial normal load.

2 Shear Load :

The load which act parallel to the cross-section is called shear load.

1. Transverse shear Load :

The load which act parallel to cross-section and passes through axis of material is called transverse shear load.

2. Eccentric shear Load :

The load which act parallel to the cross-section and away from the axis is called eccentric shear load.

- According to Time ;
  1. Static Load
     a. Dead load
     b. Gradually Applied Load
  2. Dynamic load
     a. Impact Load
     b. Fatigue Load

Stress

When external forces acts on a body it undergoes deformation and when body undergoes deformation ; molecules of the body offers some deformation to resistance ."The resistance to deformation per unit cross-sectional area is called stress."

$$\text{Stress} = \frac{\text{Resisting force}}{\text{C/S Area}},$$

for equilibrium condition,

Resiting force = External force.

- UNIT of STRESS - N/m^2.
- $1\ N/m^2 = 10^{-4}\ N/cm^2 = 10^{-6}\ N/mm^2$.
- Kilo - 10^3
  Mega - 10^6
  Crega - 10^9
  Tarro - 10^{12}

Types of stress;

1 Normal stress ;

When the applied load is perpendicular to the cross-sectional and passing through the longitudinal axis the stress induced is called normal stress.

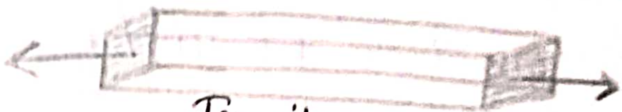
- It is denoted by 'σ'.
- $\boxed{\sigma = P/A}$
- There are some types of Normal stress;
  a. Tensile Normal stress
  b. Compressive Normal stress.

a. Tensile Normal Stress;

The stress induced in a body due to tensile force at a result of which length of body increases is called tensit normal stress.

Mathematically,

$$\sigma_t = \frac{P}{A}.$$



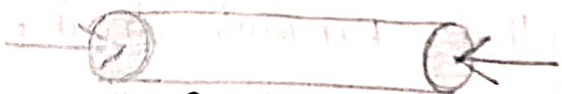
Tensile

b. Compressive Normal Stress;

The stress induced in a body due to ~~tensile~~ Compresive force at a result of which length of body decreases is called compressive normal stress,

Mathematically,

$$\sigma_c = \frac{P}{A}.$$



Compresive

Shear stress:

When two equal and opposite shear forces not in same line act on two parts of a body, then one part tends to slide over or shear from other across any section then stress develop is called shear stress.

- It is denoted by τ (tau).
- $\tau = \frac{P}{A}$.
- Unit is N/m^2.

Strain :

When a load is applied on the body, the body get deform.

The deformation produced by stress in a body is called strain.

Strain has no unit.

Type of strain;

1. Normal Strain
2. Shear strain

Normal strain

When the load is applied perpendicular to the cross-section and the deformation produce by the stress is called normal strain.

It is denoted by E.

$$\boxed{E = \frac{\text{Change in dimension}}{\text{Original dimension}}}$$

Type of Normal Strain,

1. Longitudnal strain
2. Lateral Strain
3. Volumetric Strain

   a. Tensile strain

   b. Compresive strain.

Longitudnal Strain

It Is the normal strain in the direction of applied load.

It is denoted by E_{long}.

$$\boxed{E_{long} = \frac{\delta l}{l} = \frac{l' - l}{l}}$$

Lateral Strain :

It is the normal strain in the direction perpendicular to the direction of applied load.

It is denoted by E_{lat}.

$$\boxed{E_{lat} = \frac{\delta d}{d} = \frac{d' - d}{d}}$$

Short Notes :

1. Longitudnal strain and lateral strain are opposite in nature.
2. For 3 D body, every longitudnal strain is associative with two lateral strain.
3. Under triaxial loading contition, total a strain is develop. ie 3 are longitudnal strain 6 are lateral strain.
4. In triaxial loading, total namheal strain in any one of the perpendicular direction is equal to algebric sum of longitudnal and two lateral strain in that direction.

Volumetric Strain :

It is the ratio of change in volume of body to original volume of body.

Mathematically ;

$$\varepsilon_v = \frac{\delta V}{V}$$

$$\varepsilon_v = \frac{V' - V_0}{V}$$

$$\varepsilon_v = \frac{\frac{\pi}{4} d'^2 l' - \frac{\pi}{4} d^2 l}{\pi/4 \times d^2 l}$$

$$\boxed{\varepsilon_v = \frac{d'^2 l' - d^2 l}{d^2 l}}$$

Poission's Ratio :

The ratio of lateral strain to longitudnal strain when material is stressed within the elastic limit is called poission's ratio.

- It is constant for a given material.
- It is denoted by μ or $1/m$.
- Mathematically,

$$\mu = \frac{-\varepsilon_{lat}}{\varepsilon_{long}}$$

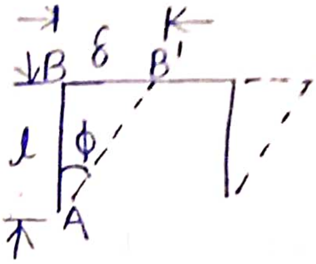
- The range of poission's ratio is "-1 to 0.5".
- In engineering, $0 \leq \mu \leq 0.5$
- In metal, $1/4 \leq \mu < 1/3$

• Metal	Poission's Ratio
Core	0
Concrete	0.1 to 0.2
Metal	1/4 to 1/3
Rubber	0.5
Parafin	0.5
Wax	0.5

Shear Strain

The strain produce due to shear force is called shear strain.

- In case of shearing load, shear strain is produce which is measured by the angle which the body displaced or distorted.



- It is denoted by 'ϕ'.
- Mathematically, $\boxed{\phi = \frac{\delta}{l} = \frac{BB'}{AB}}$

Hooks Law;

Hooks law state that, " When material is loaded upto porponality stress is directly perportional to strain."

Mathematically,

Stress ∝ Strain

Stress = Elastic Constant x Strain

$$\boxed{\text{Elastic Constant} = \frac{\text{Stress}}{\text{Strain}}}$$

There are three types of elastic constant.

1. Young's Modulus or Modulus of elasticity (E).
2. Shear modulus or modulus of rigidity (G or C).
3. Bulk Modulus (K).

Young's Modulus or Modulus Of Elasticity :

It is the ratio of Normal stress to longitudnal strain under uniaxial state of stress condition.

- It is denoted by 'E'.
- Mathematically, $\boxed{E = \frac{\text{Normal stress } (\sigma)}{\text{longitudnal strain } (\varepsilon_{long})}}$

Shear Modulus or Modulus Of Rigidity :

The ratio of shear stress to crosspondiug shear strain is called shear Modulus.

- It is denoted by G or C.
- Mathematically ; $\boxed{G = \frac{\text{Shear Stress}(\tau)}{\text{Shear Strain}(\phi)}}$

Bulk Modulus (K) :

The ratio of normal stress to volumetric strain is called bulk modulus.

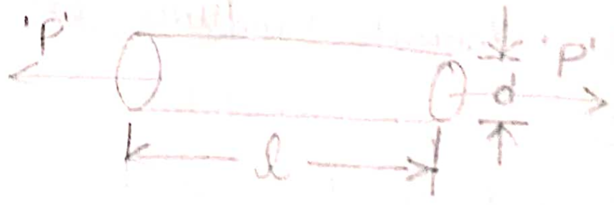
It is denoted by K.

Mathematically.

$$\boxed{K = \frac{\text{Normal stress } (\sigma)}{\text{Volumetric strain}(E_v)}}$$

Deformation due to axial load ;

Let us consider a prismatic bar of diameter 'd' and length 'l', Now consider a axial normal tensile load is acting on the body.



$$\because \sigma = \frac{P}{A}$$

and also $E_{long} = \frac{\delta l}{l}$

$$E = \frac{\sigma}{E_{long}}$$

$$E = \frac{P \times l}{A \times \delta l} \Rightarrow \boxed{\delta l = \frac{Pl}{AE}}.$$

Note:

Condition to satisfy $\delta L = \frac{PL}{AE}$ is,

- Bar should be prismatic,
- Bar should be under pure load,
- Bar should be made up of same material.

Qn. A steel wire 2 m long 3 mm in diameter is extended by 0.75 mm when a weight of 'W' is suspended from the wire. If the same weight is suspended from a brass wire 2.5 m long and 2 mm diameter, it elongate by 4.64 mm. Determine the modulus of elasticity of the brass if that steel be 2.0×10^5 N/mm^2. [SOM - live - 13]

Qn. A hollow cast iron cylinder 4 m long, 300 mm outer diameter and thickness of 50 mm is subjected to a central load on the top when standing straight. The stress produced is 75000 kN/m^2. Take young modulus for cast iron 1.5×10^8 kN/m^2.

Find (i) Longitudnal strain

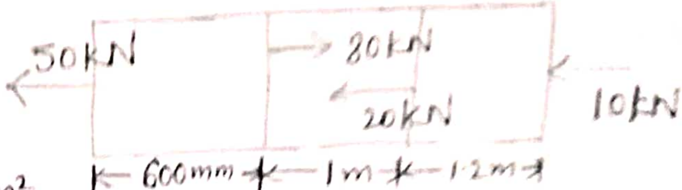
(ii) Total decreament in length. [Som - live - 13]

Principle of Superposition:

- When number of load are acting on a body, the resulting strain according to principle of superposition will be the algebric sum of strain caused by indivisual load.
- While using this principle for an elastic body which is subjected to a number of direct forces at different section along the length of the body. first free body diagram of indivisual section is drawn, then obtain deforoation of the each section.

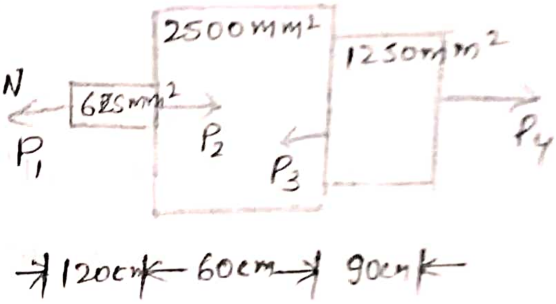
• The total deformation of the body will be equal to the algebric sum of deformation of indivisual section.

Qn. A brass bar having cross-section area of 1000 mm^2 is subjected to axial forces,
Find the total deformation of bar, Take $E = 1.05 \times 10^5$ N/mm^2.



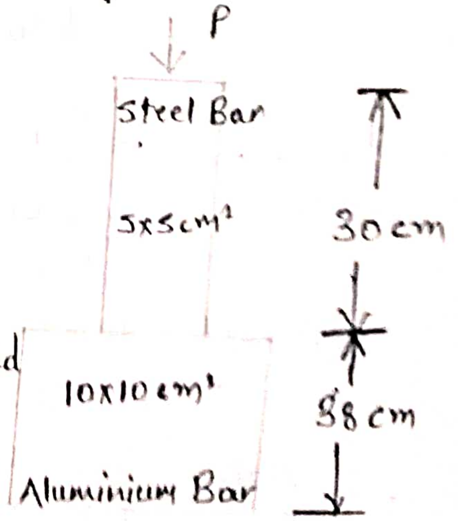
[SOM - live - 13]

Qn. A member ABCD is subjected to point load P_1, P_2, P_3 & P_4.
Calculate the force P_2 necessary for equilibrium if $P_1 = 45$ kN, $P_3 = 450$ KN and $P_4 = 130$ kN. Determine total elongation in bar. Assume
$E = 2.1 \times 10^5$ N/mm^2

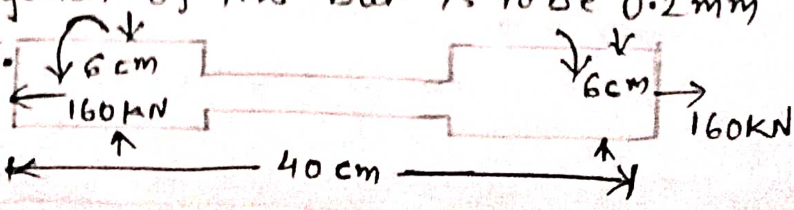


[SOM - live - 14]

Qn. A member formed by connecting a steel bar to an aluminium bar. Assuming that the bar are prevented from buckling sideways. Calculate the magnitude of force 'P' that will cause the total length of the member to decreace 0.25 mm.
The value of elastic modulus for steel and aluminium are 2.1×10^5 N/mm^2 and 7×10^4 N/mm^2. [SOM - live - 14]



Qn. The bar is subjected to a tensile load of 160 KN. If the stress in the middle portion is limited to 150 N/mm^2. Determine the diameter of middle portion. Find also the length of middle portion if the total elongation of the bar is to be 0.2 mm
Take $E = 2.1 \times 10^5$ N/mm^2.



[SOM - live - 14]

Composite Bar

For compsite bar two points are important;

a. The extension of compression in each bar is equal. Hence deformation per unit length. ie strain in each bar is equal.

$$\boxed{\frac{\sigma_1}{\sigma_2} = \frac{E_1}{E_2}}.$$

b. The total external load on composite bar is equal to the sum of the load carries by each different material.

$$P = P_1 + P_2$$

$$\boxed{P = \sigma_1 A_1 + \sigma_2 A_2}$$

Qn. A steel rod of 3cm diameter is enclosed centrally in a hollow copper tube of external diameter 5 cm and internal diadmeter 4 cm. The composite bar is then subjected to an axial pull of 45000 N. If the length of each bar is 15 cm. Find, [SOM - live - 15]

a. Stresses in the rod,

b. Load carried by each bar

$E_s = 2.1 \times 10^5 \ N/mm^2$ & $E_c = 1.1 \times 10^5 \ N/mm^2$.

Qn. A steel bar 500 mm long and 75 mm diameter is placed inside aluminium tube with 10 mm inside diameter and 100 mm outside diameter. The tube is longer by 0.2 mm than the bar. A axial compressor load of 500 kN is applied though rigid cover plate fitted at ends. Calculate the stresses in bar and tube. Take

$E_s = 200 \ kN/mm^2$ & $E_{al} = 75 \ kN/mm^2$ [SOM - live - 15]

Volumetric strain due to uniaxial load;

Consider a ractangular bar of length L, width b, and depth d, which is subjected to axial load P in direction of its length.

Let,
δl = change in length
δb = change in width
δd = change in depth

∴ Final length in bar = $l + \delta l$
.. Width .. .. = $b + \delta b$
.. depth .. .. = $d + \delta d$.

original volume of bar, $V = lbd$
Final Volume of bar, = $lbd + bd\delta l + lb\delta d + Ld\delta b$.

Chang in volume = Final Volum − original volume
= $bd\delta l + lb\delta d + ld\delta b$.

Volumetric strain,

$$E_v = \frac{\delta V}{V} = \frac{bd\delta l + lb\delta b + ld\delta b}{lbd}$$

$$= \frac{\delta l}{l} + \frac{\delta b}{b} + \frac{\delta d}{d}$$

$$= E_{long} + E_{lat} + E_{lat}.$$

∴ $E_v = E_{long} + 2E_{lat}$.

∴ $\frac{\delta d}{d} = -\mu \frac{\delta l}{l}$ or $\frac{\delta b}{b} = -\mu \frac{\delta l}{l}$.

∴ $E_v = \frac{\delta l}{l} - \mu \frac{\delta l}{l} - \mu \frac{\delta l}{l}$

$$\boxed{E_v = \frac{\delta l}{l}(1 - 2\mu)}$$

A steel bar which 4 m long, 30 mm wide and 20 mm thick and is subjected to an axial pull of 30 kN in the direction of its length. Find volumetric strain & final volume of the given bar. Take $E = 2 \times 10^5 \ N/mm^2$ & $\mu = 0.3$. [SOM - live -15]

Qn. A steel bar 300 mm long, 50 mm width and 40 mm thick is subjected to the pull of 300 kN in the direction of its ~~length~~. Determine change in volume. Take $E = 2 \times 10^8 \ N/mm^2$ & $\mu = 0.25$. [SOM - Live -15]

Volumetric strain under triaxial load;

Consider a ractangular bar of length l, width b and thickness d which is subjected to axial load. P_x, P_y & P_z in x direction, y direction and z-direction respectly.

$$\sigma_x = \frac{P_x}{bd}, \quad \sigma_y = \frac{P_y}{lb}, \quad \sigma_z = \frac{P_z}{ld}$$

$$E_v = \frac{\delta V}{V}, \quad E_x + E_y + E_z \quad - (1)$$

Strain load	x - direction	y - direction	z - direction
	σ_x / E	$-\frac{\mu \sigma_x}{E}$	$-\frac{\mu \sigma_x}{E}$
	$-\frac{\mu \sigma_y}{E}$	$\frac{\sigma_y}{E}$	$-\frac{\mu \sigma_y}{E}$
	$-\frac{\mu \sigma_z}{E}$	$-\frac{\sigma_z}{E}$	σ_z / E

$$E_x = \frac{\sigma_x}{E} - \frac{\mu \sigma_y}{E} - \frac{\mu \sigma_x}{E}$$

$$E_x = \frac{1}{E}[\sigma_x - \mu(\sigma_y + \sigma_z)]$$

$$E_y = \frac{1}{E}[\sigma_y - \mu(\sigma_x + \sigma_z)]$$

$$E_z = \frac{1}{E}[\sigma_z - \mu(\sigma_y + \sigma_x)]$$

$$E_v = E_x + E_y + E_z$$

$$\boxed{E_v = \frac{1 - 2\mu}{\ } (\sigma_x + \sigma_y + \sigma_x)}$$

Note;

- If $\mu = 0.5$, the change in volumetric strain is zero or $\sigma_x + \sigma_y + \sigma_z = 0$, but change in dimmension is not zero.
- If $\mu = 0.5$, change in volume equal to zero.
- If $\mu < 0.5$, the change in volume is equal to zero, if sum of stress in three mutual perpendicular direction is equal to zero, when, $\sigma_x + \sigma_y + \sigma_z = 0$.

Under hydrostatic state of stress condition,

$$\sigma_x + \sigma_y + \sigma_z = 0.$$

$$\because \; E_v = \left[\frac{1-2\mu}{E}\right]3\sigma$$

$$\frac{E}{3(1-2\mu)} = \frac{\sigma}{E_v}$$

$$\Rightarrow \frac{E}{3(1-2\mu)} = K$$

$$\therefore \boxed{E = 3K(1-2\mu)}$$ Relation between Young moduls & Bulk modulus.

Qn. A metallic bar 3000mm x 100 mm x 40 mm is subjected to force of 5 kN (tensile), 6 kN (tensile) and 4 kN (tensile) along x, y & z-direction respectively. Determine change in volume of the block.
Take $E = 2 \times 10^5$ N/mm² & $\mu = 0.25$. [SOM-live-15]

Qn. A metallic block 250mm x 100mm x 050 mm is subjected to force of 400 kN (tensile), 4 MN (Compression) and 2 MN (tensile) along x, y. & z direction respectively. Determine change in volume. Take $E = 2 \times 10^5$ N/mm² & $\mu = 0.25$. [SOM-live-15]

Thermal Stress:

- The stress induced due to temperature difference is called thermal stress.
- The thermal stress produced in a body when the temperature of the body is raised or lower and the body is not alloy to expand or contract freely.
- If body is allow to expand or contract freely, no stress will be set up in the body.
- Thermal stress is always normal tensile or normal compresive.

Free expantion of ~~a~~ ractangular bar:

Consider a ractangular bar of initial length 'l_i', initial thickness 't_i', and initial width 'b_i', whose temperature is increased by T°c having modulus of elasticity 'E' and coefficient of thermal expantion α and bar is free to expands.

Thermal Expantion,

$$\delta l = l_f - l_i = (\delta_{th})_x = \alpha T l$$

$$\delta b = b_f - b_i = (\delta_{th})_y = \alpha T b$$

$$\delta t = t_f - t_i = (\delta_{th})_z = \alpha T t$$

Thermal strain,

$$(\varepsilon_{th})_x = \frac{\delta l}{l} = \frac{\alpha T l}{l} = \alpha T$$

$$(\varepsilon_{th})_y = \frac{\delta b}{b} = \frac{\alpha T b}{b} = \alpha T$$

$$(\varepsilon_{th})_z = \frac{\delta t}{t} = \frac{\alpha T t}{t} = \alpha T.$$

Volumetric strain,

$$\boxed{\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z = 3\alpha T}.$$

Change in volume,

$$\varepsilon_v = \frac{\delta V}{V} = 3\alpha T, \quad \delta V = 3\alpha T V \Rightarrow \boxed{\delta V = 3\alpha T(l_i b_i t_i)}$$

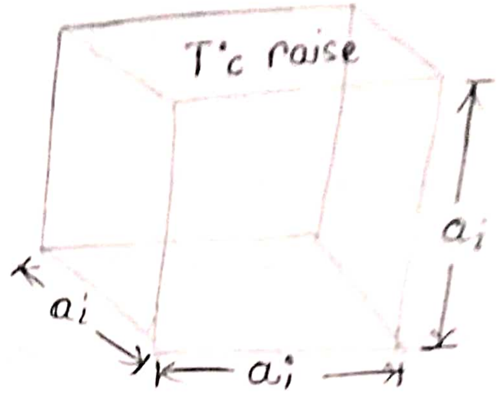
Thermal stress;

$(\sigma_{th})_x = (\sigma_{th})_y = (\sigma_{th})_z = 0.$

This is because there is no external force in any direction

ie. $P_x = P_y = P_z = 0.$

Note; For Cube



$(\delta_{th})_x = \alpha T a_i$

$(\delta_{th})_y = \alpha T a_i$

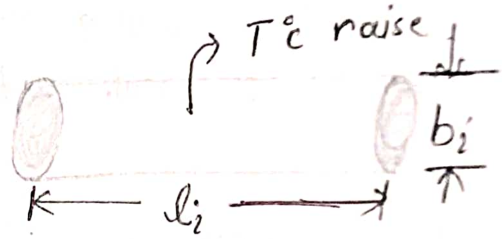
$(\delta_{th})_z = \alpha T a_i$

$(\epsilon_{th})_x = (\epsilon_{th})_y = (\epsilon_{th})_z = \alpha T$

$\epsilon_v = 3\alpha T$

$\boxed{\delta V = 3\alpha T a_i^3}.$

For Cylindrical bar,



$(\delta_{th})_x = \alpha T l_i$

$(\delta_{th})_y = (\delta_{th})_z = \alpha T d_i$

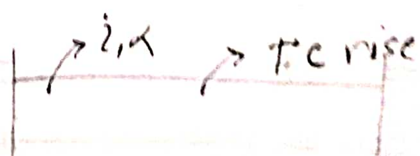
$(\epsilon_{th})_x = (\epsilon_{th})_y = (\epsilon_{th})_z = \alpha T$

$\epsilon_v = 3\alpha T$

$\boxed{\delta V = 3\alpha T \times \pi/4 \, d_i^2 l_i}.$

Completely restriction expantion in one direction,

A prismatic bar is rigidly held between two supports



$\delta l = 0$

$(\delta_{th})_x + (\delta_{axial})_x = 0$

$\alpha T l_0 + \left(\frac{-R l_0}{EA}\right) = 0$

$\frac{R l_0}{EA} = \alpha T l_0$

$\boxed{R = \alpha T E A}$

15 We know,

$$\text{Stress} = \frac{\text{load}}{\text{Area}}$$

$$(\sigma_{th})_x = \frac{R}{A} = \frac{\alpha TAE}{A}$$

$$\boxed{(\sigma_{th})_x = \alpha TE}$$

Note.

$$(\sigma_{th})_x = \pm \alpha TE$$

- When temp. rise occure, σ_{th} is compresive in nature.
- When temp. fall occure, σ_{th} is tensile in nature.
- From above equation, $\sigma_{th} \propto f(\alpha, E \,\&\, T)$. ie σ_{th} is independent of length and cross-sectional area.
- $\sigma_{th} = \pm \alpha TE$

$$(\varepsilon_{long})_x = \frac{\sigma_{th}}{E} = \frac{-\alpha TE}{E} = -\alpha T$$

$$(\varepsilon_{th})_x = \alpha T$$

$$\varepsilon_{total} = (\varepsilon_{long})_x + (\varepsilon_{th})_x = -\alpha T + \alpha T = 0.$$

- Partially restricted exantion in one direction;

A prismatic bar rigidly supported at one end and partiall restricted in other end.

$$\delta l = \lambda$$

$$(\delta_{axial})_{th} + (\delta_{th})_x = \lambda$$

$$-\frac{Rl_0}{AE} + \alpha T l_0 = \lambda$$

$$-\frac{\sigma_{th} l_0}{E} + \alpha T l_0 = \lambda$$

$$\boxed{\sigma_{th} = \pm \frac{(\alpha T l_0 - \lambda)E}{l_0} = \frac{(\delta_{th} - \lambda)E}{l_0}}$$

Note ; $\sigma_{th} = \pm \left(\frac{\delta_{th} - \lambda}{l_0} \right) E$

if +ve → temperature fall

if -ve → temperature rises

for free expantion , $\lambda = \delta_{th} \rightarrow \sigma_{th} = 0$

for completly restricted, $\lambda = 0 \rightarrow \sigma_{th} = \pm \alpha T E$.

Qn; A rod is 2m long at a temperature of 10°C. Find the expantion of rod When the temperature is raised to 80°C. If this the ~~temperature~~ expantion is prevented. Find the stress induced in the material of rod. Take $E = 1 \times 10^5$ MN/m^2 and $\alpha = 0.000012$ per degree centrigrade.

[SoM - live - 20]

Qn. A steel rod of 3 cm diameter and 5m long is connected to two grips and the rod is maintained at the temperature of 95°C. Determine the stress and pull exerted When the temperature fall to 30°C if

a. the ends donot yeild [SoM - live - 20]

b. the end yield by 0.12 cm

Take $E = 2 \times 10^5$ MN/m^2, $\alpha = 12 \times 10^{-6}$ /°C.

Qn. A Steel bar of diameter 30 mm and length 2m is Subjected to pull of 10kN. Determine the total Stress When temperature rise is 20°C. Take $E = 2 \times 10^{11}$ N/m^2

$\alpha = 12 \times 10^{-6}$ /°C assume both ends is fixed.

[SoM / live - 20]

Mechanical Property of metals;

Strength : The ability of material to resist the external applied force without breaking or yielding is called strength.

Stiffness : The ability of material to resist deformation under stress is called stiffness.

Elasticity :

- It is the property of material to regain its original shape after deformation when the external force removed.
- This property is desirable for material used in tool & machine.
- It may be known that still is more elastic than rubber.

Plasticity :

- This property of material which retained the deformation produce under load permanently is called plasticity.
- This property of material is necessary forging is stamping image on coin and in ornamental work.

Ductility :

- The property of material enbling it to be drawn into wire with the application of tensile force is called ductility.
- Ductile material commonly used in engineering practice are mild steel copper aluminium, nickel, zink & Lead.

Brittleness :

- The property of material opposite to ductility is called brittleness.
- It is the property of breaking of material with little permanent distortion.
- Cast iron is brittle.

Toughness :

- It is the property of material to resist fracture due to high impact load like hammer blow.
- The toughness of material decrease when it is heated this property is desirable in a part subjected to shock for impact loadin

Str

Malleability:

- It is special case of ductility which permit material to be rolled or hammered into thin sheet.
- A malleable material should be plastic but it is not essential to be so strong.
- The malleable material ~~should be so strong~~ commonly used in engineering practice are lead soft steel wrote iron copper aluminium.

Creep:

- When a part is subjected to constant stree at high temperature for a long period of time it will undergoes slow and permanent deformation called creep.
- This property is considered is designing intenal combustion engine, boiler and turbine.

Fatigue:

- When a material is subjected to respected stress it failure of material is called fatigue.
- The failure is caused by means of progressive crack deformation which are usually fine and microscopic.

Hardness:

- It is very important property of metal & has wide varieties of meaning it embaraces many different properties such as resistance to wear, scaratching, deformation, machine ability etc.
- It also means the ability of material to cut another metal.
- The hardness is usually expressed in number which depend upon the method of ~~the~~ making the test.

Strain Energy

The energy absorb by the member when works done by the load deform the member is called strain energy.

For safe condition, so that failure not occure, strain energy of bar (Interal energy) is equal to workdone by load (External energy) is equal to area under the force - extension curve.

Resilience

The strain energy stored by the body within elastic limit, when loaded externally is called resilience.

Let us consider a bar of cross-sectional area 'A' and length l and subjected to load 'W'.

Suppose this load extends the bar by an amount δl and produce maxm stress σa.

$\because$ Strain energy in bar = Work done by load

$$U = \frac{1}{2} W \cdot \delta l$$

$$U = \frac{1}{2} W \cdot \frac{Wl}{AE}$$

$$\boxed{U = \frac{W^2 l}{2AE}}$$ Nm or Joule

Then, $$U = \frac{\sigma a^2 A^2 l}{2AE}$$

$$\Rightarrow \boxed{U = \frac{\sigma a^2 V}{2E}}$$ Joule

Again,

$$U = \frac{1}{2} \times \sigma a \times \frac{\sigma a}{E} \times A \times l$$

$$\Rightarrow \boxed{U_o = \frac{1}{2} \sigma a \times E_{long} \times AL}$$

Proof Resilience:

- The maximum energy absorbed by a member within elastic region is called Proof Resilience.
- The maximum resilience is also called proof resilience.
- The energy absorbed by the member at elastic point is known by proof resilince.
- Proof resilience, U_P,

$$\boxed{U_P = \frac{1}{2E} \sigma_p^2 V} \quad \text{Joule}$$

Modulus of Resilince

Proof resilince of material per unit volume is called modulus of resilince.

$$\text{Modulus of resilience} = \frac{\text{Proof Resilince}}{\text{Volume}}$$

$$MOR = \frac{U_P}{V} = \frac{1 \times \sigma_p^2 \cancel{V}}{2E \times \cancel{V}} = \frac{\sigma_p^2}{2E},$$

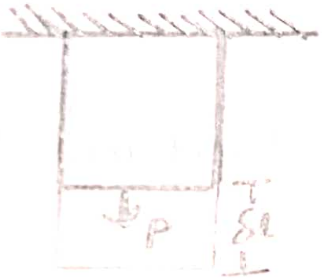
$$MOR = \frac{\frac{1}{2} W_P \times \delta l_P}{V} = \frac{1}{2} \times \frac{W_P}{A} \times \frac{\delta l_P}{l}$$

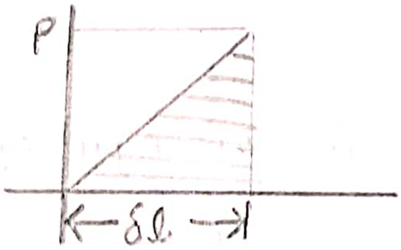
$$\boxed{\text{Modulus of Resilince} = \frac{1}{2} \times \text{Stress} \times \text{longitudnal strain}}$$

Strain Energy store in a body when load is applied gradually;

A body is said to be acted upon by a gradually applied load is the load increases from zero and and reacts its final value stepwise.

Consider a uniform bar subjected to gradually load. As load increases from zero to P, deformation of bar also increas from 0 to δl.





Work strain energy of bar = Workdone by load

$$U = \frac{1}{2} P \, \delta l$$

$$U = \frac{1}{2} P \times \frac{Pl}{AE}$$

$$U = \frac{P^2 l}{2AE}$$

$$U = \frac{\sigma^2 A^2 l}{2AE}$$

$$\boxed{U = \frac{\sigma^2 V}{2E}}$$

Strain energy store in the body when load is applied suddenly;

When the load is applied suddenly not stepwise on object or body is called suddenly applied load.

Let, the load 'W' is applied suddenly & maximum stress thus produce be σ_{su} and extension being δl.

then,

$\because$ Strain energy of bar = Workdone by load

$$\frac{1}{2}\sigma_{su} \times A \times \delta l = W \times \delta l$$

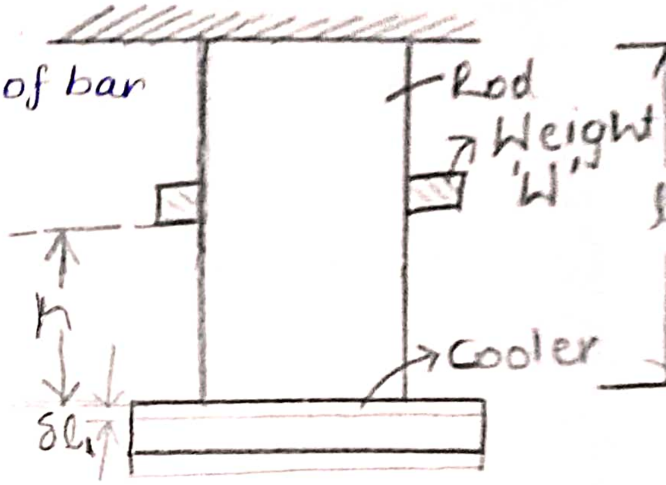
$$\boxed{\sigma_{su} = \frac{2W}{A}}$$

" The stress (σ_{su}) due to sudden applied load is double of gradually applied load."

Strain energy ~~store~~ store in the body due to impact load;

- The load which fall from a height or strike on the body with certain momentum is called impact load.

Consider a Weight 'W' falling through a Height 'h' on a cooler fitted on the rod which of length 'l' and cross-section 'A'.



External Workdone = Strain energy of bar

$$W(h+\delta l_i) = \frac{1}{2}\sigma_i A\,\delta l_i$$

$$W\left(h+\frac{Wl}{AE}\right) = \frac{1}{2}\sigma_i A\,\delta l_i$$

$$\Rightarrow W\left(h+\frac{\sigma_i l}{E}\right) = \frac{1}{2}\sigma_i A\frac{Wl}{AE}$$

$$= Wh + \frac{W\sigma_i l}{E} = \frac{\sigma_i\,\sigma_i \times A \times l}{2E}$$

$$\Rightarrow Wh + \frac{W\sigma_i l}{E} = \frac{\sigma_i^2 A l}{2E}$$

$$\Rightarrow \frac{\sigma_i^2 A l}{2E} - \frac{W\sigma_i l}{E} - Wh = 0$$

$$\Rightarrow \sigma_i = \frac{\frac{Wl}{E} \pm \sqrt{\frac{W^2 l^2}{E^2} - 4\frac{Al}{2E}\times(-Wh)}}{2\frac{Al}{2E}}$$

$$\sigma_i = \frac{\frac{Wl}{E} \pm \frac{Wl}{E}\sqrt{1+\frac{2WhAl}{E}\times\frac{E^2}{W^2 l^2}}}{Al/E}$$

$$\Rightarrow \boxed{\sigma_i = \frac{W \pm W\sqrt{1+\frac{2hAE}{Wl}}}{A}}$$

Note;

If δl_i is negligible as compared to h,

Then,
$$Wh = \frac{\sigma_i^2 A l}{2E}$$

$$\Rightarrow \boxed{\sigma_i = \sqrt{\frac{Wh\,2E}{Al}}}$$

If $h = 0$ → Sudden Applied case

$$\boxed{\sigma_i = \frac{2W}{A}}$$

Qn. A steel wire 2m long and 3 mm in diameter is extended by 0.75 mm when a weight w is suspended from the wire. If the same weight is suspended from the wire, 2.5 m long and 2 mm diameter, it is elongated by 4.64 mm. Determine the modulus of elasticity of brass, if that of steel be 2.0×10^5 mm².

Given,

$l_s = 2m = 2 \times 1000$ mm

$l_b = 2.5m = 2.5 \times 1000$ mm

$d_s = 3$ mm

$d_b = 2$ mm

$E_s = 2 \times 10^5$ N/mm², $E_b = ?$

$\delta l_s = 0.75$ mm

$\delta l_b = 4.64$ m

$P_s = P_b = \omega$

For steel,
$$\delta l_s = \frac{P_s l_s}{A_s E_s}$$

$$\frac{75}{100} = \frac{\omega \times 2000}{\frac{\pi}{4} 9 \times 2 \times 10^5}$$

$$\omega = \frac{75 \times 9 \pi}{4}$$

For brass,
$$\delta l_b = \frac{P_b l_b}{A_b E_b}$$

$$E_b = \frac{75 \times 9 \times \pi \times 2.5 \times 1000 \times 4}{4 \times \pi \times 4 \times 4.64}$$

$$= 90921.34 = 0.90 \times 10^5 \text{ N/mm}^2$$

Qn. A hollow cast iron cylinder 4 m long, 300 mm outer diameter and thickness of 50 mm is subjected to a control load on the top when standing straight. The stress produced is 75000 kN/m². Take young's modulus for cast iron 1.5×10^8 kN/m². Find a. Longitudnal strain & b. total decreas in length.

Given,

$D = 300\ mm = 0.3\ m$

$T = 50\ mm = 0.05\ m$

$l = 4\ m$

$\sigma = 75000\ kN/m^2$

$E_s = 1.5 \times 10^8\ kN/m^2$

Inner diameter, $d = D - 2t$
$= 0.3 - 0.1 = 0.2\ m$

longitudnal strain, $\epsilon_{long} = \frac{\sigma}{E} = \frac{\cancel{75000} \times \cancel{10^3} \times 10}{\cancel{1.5} \times 10^{\cancel{4}} \times \cancel{10^3}} = \underline{\underline{5 \times 10^{-4}}}\ m$

Decrease in length,

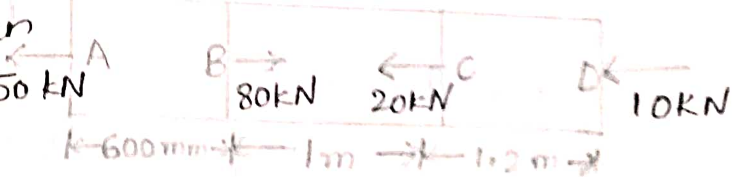
$\epsilon_l = \frac{\delta l}{l} \Rightarrow 0.0005 \times 4 = \delta l$

$\delta l = 0.002\ m = \underline{\underline{2\ mm}}$

Qn. A brass bar having cross-sectional area of 1000 mm² is subjected to axial forces as shown in figure;

Find the total deformation of bar

Take $E = 1.05 \times 10^5$ N/mm².

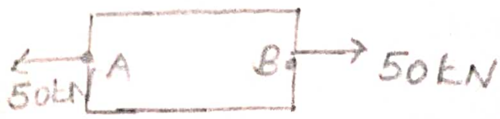


Given, area, $A = 1000\ mm^2$

$E = 1.05 \times 10^5\ N/mm^2$

Let, dl = total deformation of the bar

For Part AB



$P_{AB} = 50\ kN$, $l_{AB} = 600\ mm$

$\delta l_{AB} = \frac{50 \times \cancel{1000} \times 600}{\cancel{1000} \times 1.05 \times 10^5} = \frac{300 \times 10^{-5}}{1.05} = 2.857 \times 10^{-5}$

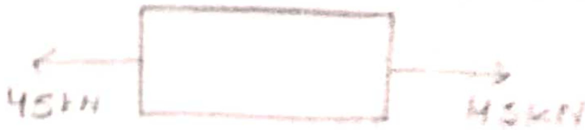
$= \underline{\underline{0.2857\ mm}}$

longation, δl.

For Part AB,

$$\delta l_{AB} = \frac{45\times10^3\times1200}{625\times2.1\times10^5}$$

$$= 0.4114 \text{ mm}$$



For Part BC,

$$\delta l_{BC} = \frac{320\times10^3\times600}{2500\times2.1\times10^5}$$

$$= 0.3657 \text{ mm}$$

For Part CD,

$$\delta l_{CD} = \frac{130\times1000\times900}{1250\times2.1\times10^5}$$

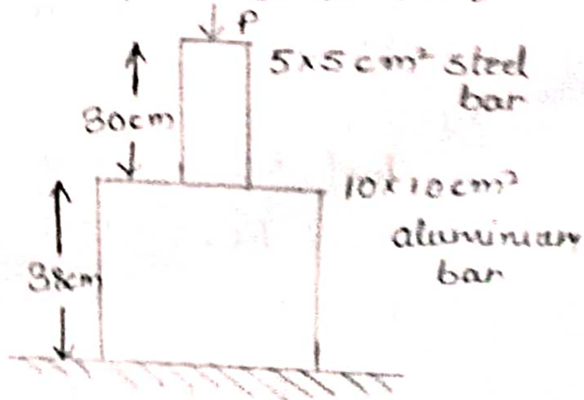
$$= 0.4457 \text{ mm}$$

Total elongation, $\delta l = \delta l_{AB} + \delta l_{BC} + \delta l_{CD}$

$$= 0.4144 \pm 0.3657 + 0.4457$$

$$= \underline{\underline{0.4944 \text{ mm}}}$$

Qn. A member formed by connecting a steel bar is shown in figure. Assuming that the bar are prevented from buckling sidways. Calculate the magnitude of force 'P' that will cause the total length of the member to decrease 0.25 mm. The value of elastic modulus for steel and aluminium are 2.1×10^5 N/mm² and 7×10^4 N/mm² respectively.



Give,

$L_S = 30 = 300$ mm

$A_S = 25\text{ cm}^2 = 2500\text{ mm}^2$

$E_S = 2.1\times10^4$ N/mm²

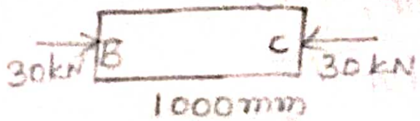
$L_A = 38 = 380$ mm

$A_A = 100\text{ cm}^2 = 10000\text{ mm}^2$

$E_A = 7\times10^4$ N/mm²

P be the load exerted.

For Part BC



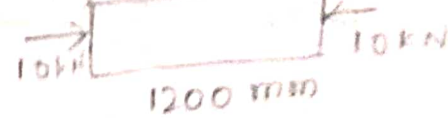
$P_{BC} = 30kN$

$L_{BC} = 1000mm$

$$\delta l_{BC} = \frac{30 \times 1000 \times 1000}{1000 \times 1.05 \times 10^5} = \frac{3}{105} \times 10^{-3} = 0.02857 \times 10^{-3}$$

$$= 0.2857\ mm$$

For Part CD



$P_{CD} = 10 \times 1000\ N$

$L_{CD} = 1200\ mm$

$$\delta l_{CD} = \frac{10 \times 1000 \times 1200}{1000 \times 1.05 \times 10^5} = \frac{120}{105} \times 10^{-5} = 1.14285$$

$$= 0.0114\ mm$$

$\because$ Total Elongation of bar $\delta l = \delta l_{AB} + \delta l_{BC} + \delta l_{CD}$

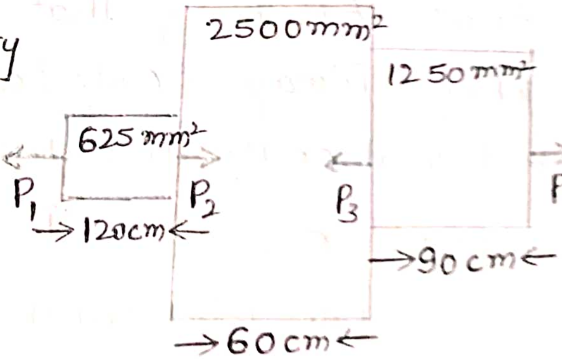
$$= 0.2857 - 0.2857 - 0.0114$$

$$= -0.0114\ mm$$

Qn.

A member ABCD is subjected to point load P_1, P_2, P_3 & P_4 as shown in figure,

Calculate the force P_2 neccesary for equilibrium if $P_1 = 45kN$, $P_3 = 450\ kN$ and $P_4 = 130kN$. Determine total elongation in bar. $E = 2.1 \times 10^5\ N/mm^2$



Given,

Part AB, Area $A_1 = 625 mm^2$, Length $l_1 = 120cm = 1200mm$

Part BC, Area $A_2 = 2500 mm^2$, Length $l_2 = 60\ cm = 600mm$

Part CD, Area $A_3 = 1250 mm^2$, Length $l_3 = 90\ cm = 900\ mm$

$\because P_1 = +45\ kN$, $P_3 = +450kN$, $P_4 = -130kN$

$P_2 = -365\ kN$

$\because \delta l = \delta l_S + \delta l_A$

$$-0.25 = \frac{-P \times 300}{2600 \times 2.1 \times 10^5} + \frac{-P \times 320}{10000 \times 7 \times 10^4}$$

$$0.25 = P\left[\frac{300 \times (1000 \times 7 \times 10^4) + 320(2600 \times 2.1 \times 10^5)}{2500 \times 2.1 \times 10^5 \times 10000 \times 7 \times 10^4}\right]$$

$$P = 224.37\ \underline{kN}$$

Qn. The bar shown in figure is subjected to a tensile load of 160 kN. If the stress in the middle portion is limited to 150 N/mm². Determine the diameter of middle portion. Find also the length of the middle portion if the total elongation of the bar is to be 0.2 mm. Take $E = 2.1 \times 10^5$ N/mm².

Longitudnal strain in middle portion, 40 cm

$$\varepsilon_{long} = \frac{\sigma}{E} = \frac{\cancel{150}^{5}}{\cancel{2.1}_{7} \times 10^5} = \underline{0.71 \times 10^{-5}}\ mm$$

Elongation in first portion,

$$\delta l_1 = \frac{PL}{AE} = \frac{16\cancel{0} \times 10^{\cancel{3}} \times L \times 4}{\pi \times 36 \times 2.1 \times 10^{\cancel{5}}}$$

$$\Rightarrow 0.00084\ L \quad -(i)$$

Elongation in Last portion, $\delta L_2 = 0.00084L \quad -(ii)$

Total elongation, $\delta l = 2$

$$(0.00084 \times 2)L + 0.71 \times 10^{-5} = 0.2$$

$$\Rightarrow L = \frac{0.00349}{0.0016\ 8} = \underline{2.077}\ mm$$

$\because$ Length of middle portion;

$$400 - 4.154 = 395.8\ mm$$

$$= 39.58\ cm$$

$\because$ $\epsilon_{long} = \frac{\delta l}{l}$

$\delta l = \epsilon_{long} \times l = 395.8 \times 0.7 \times 10^{-5}$

$= 0.0027\ mm$

$\because$ $\delta l = \frac{P \times L}{A \times E}$

$0.0027 = \frac{160 \times 395.8 \times 4}{\pi \times d^2 \times 2.1 \times 10^5}$

$= d^2 = \frac{160 \times 395.8 \times 4}{0.0027 \times 3.14 \times 2.1 \times 10^5}$

$= d = \sqrt{142.27} = 11.92\ mm$

$\therefore$ Length of middle portion $= 395.8\ mm$

Diameter of middle portion $= 11.92\ mm$

Qn. A steel rod of 3 cm diameter is enclosed centrally in a hollow copper tube of external diameter 5 cm and internal diameter 4 cm. The composite bar is then subjected to an axial pull of 4500 N. If the length of each bar is 15 cm. Find,

a. The stress in the rod.

b. Load carried by each bar.

$E_s = 2.1 \times 10^5\ N/mm^2$; $E_c = 1.1 \times 10^5\ N/mm^2$.

Given, $D_s = 3cm = 30\ mm$

$A_s = \frac{\pi}{4} \times 30^2 = 706.86\ mm^2$

External diameter of copper $= 5cm = 50\ mm$

Internal diameter of copper $= 4cm = 40\ mm$

$\therefore$ Area of copper, $A_c = \frac{\pi}{4}(2500 - 1600) = \frac{\pi}{4} 900 = 706.86\ mm^2$

Stresses in tube and rod;

σ_s = Stress in steel

P_s = Load Carried by steel rod

σ_c = Stress in copper

P_c = Load carried by copper rod

Strain in steel = strain in copper

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_s}$$

$$\Rightarrow \sigma_s = \frac{2.1\times10^5}{1.1\times10^5}\sigma_c$$

$$\sigma_s = 1.909\,\sigma_c \quad -(1)$$

Total Load = Load on steel + Load on copper

$$P = \sigma_s \times A_s + \sigma_c \times A_c$$

$$45000 = 1.909\times706.86\times\sigma_c + 706.86\,\sigma_c$$

$$45000 = 706.86\,\sigma_c\,[1.909+1]$$

$$45000 = 2056.25\,\sigma_c$$

$$\sigma_c = \frac{45000}{2056.25} = 21.88\ N/mm^2$$

$$\sigma_s = 1.909\,\sigma_c = 1.909\times21.88 = 41.77\ N/mm^2$$

$\therefore$ Load by steel bar $= \sigma_s \times A_s = 41.77\times706.86 = \underline{\underline{29525.54N}}$

Load by steel bar $= \sigma_c \times A_c = 21.88\times706.85 = 15466.1N$

Qn. A steel bar 500 mm long and 75 mm diameter is pla[ced] inside aluminium tube with 80 mm inside diameter and 100 m[m] outside diameter. The tube is longer 0.2 mm than the bar. A axia compressive load of 500 kN is applied through rigid cover plate fitted at ends. Calculate the stresses in bar and tube. Take $E_s = 200\,kN/mm^2$ and $E_{Al} = 75\,kN/mm^2$

Length of steel, $l_s = 500$ mm

Length of aluminium tube, $l_{al} = 500 + 0.2 = 500.2$ mm

Area of tube, $A_{al} = \frac{\pi}{4}(100^2 - 80^2) = 2827.47\ mm^2$

Area of bar, $A_s = \frac{\pi}{4}(75)^2 = 4417.86\ mm^2$

$E_s = 200\ kN/mm^2$, $E_{al} = 75\ kN/mm^2$

Let Δl decreases in length of ~~500~~ aluminium,

$$\Delta l = \frac{P_1 l_1}{A_1 E_1}$$

$$0.2 = \frac{P_1 \times 500.2}{2827.47 \times 75}$$

$$P_1 = \frac{0.2 \times 75 \times 2827.47}{500.2} = 84.79\ kN$$

Let stress in tube, $= \sigma_{al}$

Stress in bar $= \sigma_s$

Strain in bar = Strain in tube

$$\frac{\sigma_s}{E_s} = \frac{\sigma_{al}}{E_{al}}$$

$$\frac{\sigma_s}{200} = \frac{\sigma_{al}}{75}$$

$$\sigma_s = 2.67\,\sigma_{al}$$

Remaining load $\Rightarrow 415.22 = \sigma_s A_s + \sigma_{al} A_{al}$

$$415.22 = 2.67 \times 4417.86 \times \sigma_{al} + \sigma_{al} \times 2827.47$$

$$\sigma_{al} = \frac{415.22}{14623.15} = 0.0283\ kN/mm^2$$

$\therefore \sigma_s = 2.67 \times 0.0283 = 0.0758\ kN/mm^2$

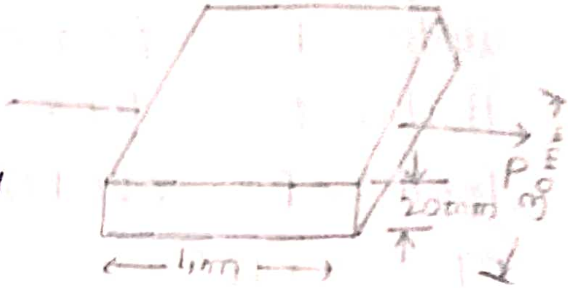
86 m^2

2. A steel bar which 4 mm long, 30 mm wide and 200mm thick and is subjected to an axial pull of 30 kN in the direction of its length. Find volumetric strain and final volume of given bar. Take, $E = 2\times10^5\ N/mm^2$ and $\mu = 0.3$.

Length, $l = 4000\ mm$

Width, $b = 30\ mm$

Thick, $t = 20\ mm$



Load $P = 30\ kN = 30\times10^3\ N$

$E = 2\times10^5\ N/mm^2$

$\mu = 0.3$

$\because$ original volume $= l\times b\times t = 4000\times30\times20 = 2400000\ mm^3$

Stress, $\sigma = \frac{Load}{Area} = \frac{\cancel{30}\times10^3}{20\times\cancel{30}} = 50\ N/mm^2$

$\epsilon_{long} \Rightarrow \frac{\delta l}{l} = \frac{\sigma}{E}$

$\frac{\delta l}{4000} = \frac{50}{2\times10^5}$

$\delta l = \frac{\cancel{50}\times\cancel{4000}}{\cancel{2}\times\cancel{10^5}} = 1\ mm$

Volumetric strain,

$$e_v = (1-2\mu)\frac{\delta l}{l}$$

$$= (1-2\times0.3)\frac{1}{4000}$$

$$= 0.4\times\frac{1}{4000} = 0.0001$$

$\because E_v = \frac{\delta V}{V} \Rightarrow 0.0001 = \frac{\delta V}{2400000}$

$\delta V = 2400000\times0.0001 = 240\ mm$

$\delta V =$ Final volume $-$ Original volume

Final volume $=$ Original volume $+ \delta V$

$= 2400000 + 240$

$= 2400240\ mm^3$

Qn. A steel bar 300 mm long, 50 mm width and 40 mm thick is subjected to a pull of 300 kN in the direction of its length. Determine change in volume. Take $E = 2 \times 10^5$ N/mm and $\mu = 0.25$.

length, l = 300 mm
width, b = 50 mm
thick, t = 40 mm
load, P = 300 kN
$E = 2 \times 10^5$
$\mu = 0.25$

original volume $= 300 \times 50 \times 40 = 600000 \text{ mm}^3$

$$\because \delta l = \frac{Pl}{AE} = \frac{300 \times \cancel{10^3} \times 3\cancel{00}}{2000 \times 2 \times \cancel{10^5}} = \frac{9\cancel{00}}{40\cancel{00}} = 0.225 \text{ mm}$$

Volumetric strain

$$E_v = (1 - 2\mu)\frac{\delta l}{l}$$

$$= (1 - 2 \times 0.25)\frac{0.225}{300}$$

$$= \frac{0.5 \times 0.225}{300} = 0.000375$$

$$\frac{\delta V}{V} = 0.000375$$

$$\delta V = 0.000375 \times 600000$$

$$= 225 \text{ mm}^3$$

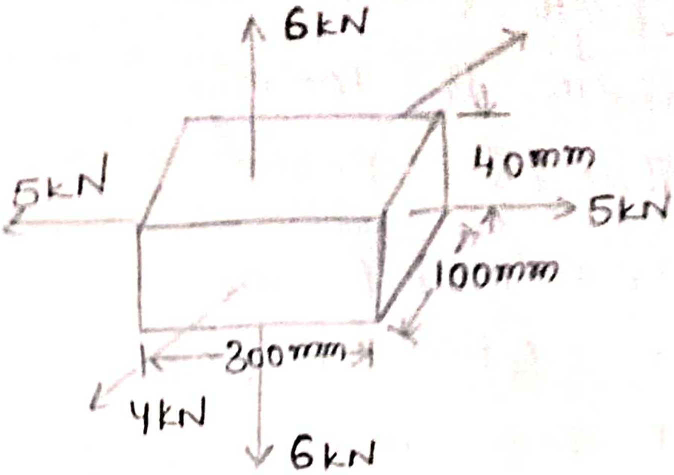
Qn. A metallic bar 3000 mm × 100 mm × 40 mm is subjected to force of 5 kN (tensile), 6 kN (tensile) and 4 kN (tensile) along x, y & z direction respectively. Detemine change in volume of the block. Take $E = 2 \times 10^5$ N/mm² and $\mu = 0.25$.

5.86 mm²

given, length, $l = 300\,mm$

Width, $b = 100\,mm$

thick, $t = 40\,mm$



$P_x = 5kN = 5 \times 10^3 N$

$P_y = 6kN = 6 \times 10^3 N$

$P_z = 4kN = 4 \times 10^3 N$

$E = 2 \times 10^5 N/mm^2$, $\mu = 0.25$

Stress along x-direction,

$$\sigma_x = \frac{5 \times 10^8}{40 \times 100} = 1.25 N/mm^2$$

Stress along y-direction,

$$\sigma_y = \frac{6 \times 10^3}{300 \times 100} = 0.2 N/mm^2$$

Stress along z-direction,

$$\sigma_z = \frac{4 \times 10^3}{300 \times 40} = 0.33 N/mm^2$$

Under triaxial loading condition,

Volumetric strain is,

$$\varepsilon_v = \frac{\delta V}{V} = \left(\frac{1-2\mu}{E}\right)(\sigma_x + \sigma_y + \sigma_z)$$

$$\Rightarrow \frac{\delta V}{300 \times 100 \times 40} = \frac{0.5}{2 \times 10^5} \times 1.78$$

$$\Rightarrow \delta V = \frac{0.5 \times 1.78 \times \overset{6}{\cancel{12}}\cancel{00000}}{\cancel{2 \times 10^5}}$$

$$= 5.34\ mm^3$$

Qn. A metallic block 250mm × 100mm × 50mm is subjected to force of 400kN (tensile), 4 MN (compressive) & 2MN (Tensile) along x, y and z direction respectively. Find change in volume. Take $E = 2 \times 10^5 N/mm^2$, $\mu = 0.25$.

Length, l = 250 mm

Width, b = 100 mm

Thick, t = 50 mm

$P_x = 400 \text{ kN} = 400 \times 10^3 \text{ N}$

$P_y = -4 \text{ MN} = -4 \times 10^6 \text{ N}$

$P_z = 2 \text{ MN} = 2 \times 10^6 \text{ N}$

$E = 2 \times 10^5 \text{ N/mm}^2$, $\mu = 0.25$.

Stress along x-direction,

$$\sigma_x = \frac{400 \times 10^3}{100 \times 50} = 80 \text{ N/mm}^2$$

Stress along y-direction,

$$\sigma_y = -\frac{4 \times 10^6}{250 \times 100} = -160 \text{ N/mm}^2$$

Stress along z-direction,

$$\sigma_z = \frac{2 \times 10^6}{250 \times 50} = 160 \text{ N/mm}^2$$

Volumetric strain under triaxial loading,

$$e_v = \frac{\delta V}{V} = \left(\frac{1 - 2\mu}{E}\right)(\sigma_x + \sigma_y + \sigma_z)$$

$$\Rightarrow \delta V = \frac{0.5}{2 \times 10^5} \times 80 \times 250 \times 100 \times 50$$

$$\Rightarrow \delta V = 250 \text{ mm}^2$$

Qn. A rod is 2m long at a temperature of 10°c. Find the expantion of rod, when the temperature is raised to 80°c. If this expansion is prevented. Find the stress induced in the material of rod. Take $E = 1 \times 10^5 \text{ MN/m}^2$ and $\alpha = 0.000012$ per degree C.

Length, $l = 2\,m$

$T_i = 10^\circ C$

$T_h = 80^\circ C$, $T = T_h - T_i = 70^\circ$

$E = 1 \times 10^5\ MN/m^2$

$\alpha = 0.000012\ /^\circ c$

∵ Thermal expansion, $\delta_{th} = \alpha T l = 0.000012 \times 70 \times 2$

$= 0.00168\ m$

$= 1.68\ mm$

∵ Thermal Stress, $\sigma_{th} = -\alpha T E$

$= 0.000012 \times 70 \times 1 \times 10^5$

$= 84\ MN/m^2$

Qn. A steel rod of 3cm dia~~gram~~ and 5m long is connected to two of rips and the rod is maintained at temperature of $95^\circ C$. Determine the ~~load~~ stress and pull exerted when the temperature fall to $30^\circ C$ if

a. the ends donot yield

b. the ends yield by 0.12 cm.

Take $E = 2 \times 10^5\ MN/m^2$, $\alpha = 12 \times 10^{-6}/^\circ c$.

Given, $l = 5\,m = 5000\,mm$

diameter, $d = 3\,cm = 30\,mm$

area $= \frac{\pi}{4} 30^2 = 225\pi\ mm^2$

$T_i = 95^\circ C$, $T_f = 30^\circ c$. $T = T_i - T_f = 65^\circ C$

$E = 2 \times 10^5\ MN/m^2$, $\alpha = 12 \times 10^{-6}\ /^\circ c$.

a. When ends donot yield (completly resistricted)

$\sigma_{th} = \alpha T E = 12 \times 10^{-6} \times 65 \times 2 \times 10^{11} = 156 \times 10^6\ N/m^2$

$\sigma_{th} = 156\ N/mm^2$

pull in rod,

$R = \sigma_{th} \times A$

$= 156 \times 225\pi$

$R = 1102699\ N$

b. When ends yield by 0.12 (partially ristricted)

$\lambda = 0.12\ cm = 1.2\ mm$

$$\sigma_{th} = \frac{(\alpha T l - \lambda)E}{l}$$

$$\sigma_{th} = \frac{(12\times10^{-6} \times 65 \times 5000 - 1.2) \times 2\times10^{5}}{5000}$$

$\sigma_{th} = 108\ N/mm^2$

pull in rod, R

$R = \sigma_{th} \times A$

$R = 108 \times 225\pi = 76340.7\ N$

Qn. A steel bar of diameter 30mm and length 2m is subjected to pull of 10kN. ~~Diameter~~ Determine the total stress when temperature rise in 20°c. Take. $E = 2\times10^{11}\ N/m^2$; $\alpha = 12\times10^{-6}/°c$. assume both ends are fixed.

Diameter, $d = 30mm$, area, $A = \frac{\pi}{4}(30)^2 = 225\pi\ mm^2$

length $= 2m = 2000mm$

load, $P = 10kN = 10\times10^{3}\ N$, $E = 2\times10^{11} N/m^2 = 2\times10^{5} N/mm^2$

$\alpha = 12\times10^{-6}/°c$ and $T = 20°c$.

Stress due to applied load,

$$\sigma_a = \frac{P}{A} = \frac{10^4}{225\pi} = 14.147 \, N/mm^2$$

Stress due to temperature rise;

$$\sigma_{th} = \alpha T E$$

$$= 12 \times 10^{-6} \times 20 \times 2 \times 10^5 = 48 \, N/mm^2$$

Total stress, $\sigma = \sigma_a + \sigma_{th}$

$$= 14.147 - 48 = -33.853 \, N/mm^2$$

$$\boxed{\sigma = 33.853 \, N/mm^2}.$$

1

UNIT-2 Shear Force and Bending Moment

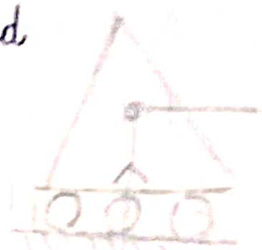
Support and calculation of support :

a. Simple support
   - Roller support
   - Hinge support

b. Fixed support

Roller Support :

In roller support, the verticle motion is ristricted, hence there is only one support reaction.



Hinge Support :

In Hinge support, the vertical and horizontal motion is ristricted, hence there is two support reaction.

Fixed support :

In Fixed support, all three motion are restricted and hence three support reaction.

Note :

- The no. of reaction ~~by~~ any support is equal to no. of motion restricted by that support.
- The no. of equation for statical equilibrium,

$$\Sigma F_x = 0, \Sigma F_y = 0, \Sigma M = 0.$$

Beam:

- Beam is the structural member which is subjected to tranverse shear load during its functionality.
- Due to trannsverse shear load, beam are subjected to variable bending moment and variable shear force.

- To know the type of variation and maximum value of shear force and bending moment, S.F.D and B.M.D are to be drawn.
- S.M.D. and B.M.D. play an important role in design of beam and shaft based on strength and rigidity critaria.

Typ

Types of Beam:

1. Cantileve Beam:

A beam whose one end is fixed and other end is free is called cantilever beam.

2. Simply supported Beam:

A beam is supported or resting freely on the support at its both ends is known as simply supported beam.

3. ~~Fixed~~ Over hanging beam:

If ~~is~~ the ends portion of a beam is extended beyonds the supported (simply), such beam is called overhanging beam.

4. Fixed Support:

A beam whose both end are fixed or built in wall is known as fixed beam. It is also called encasted or built-in beam

5. Continuous Beam:

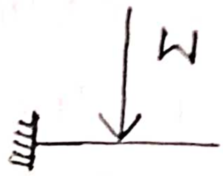
A beam which produced more than two simply support is called continous beam.

Type of load:

1. Concentrated or Point Load
2. Uniformly distributed Load
3. Uniformly varying distributed Load
4. Cocentrated moment.

Concentrated or Point Load

A concentrated or poiltt load, which is concentrated to act at a point.

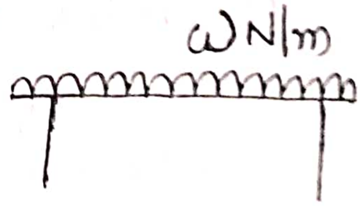


- 'W' be the point load.

Uniformly Distributed Load

A uniformly distributed is one which is speard over a beam in such a manner that rate of loading as is uniform along the length.

- The rate of loading is expressed as W N/M.
- For solving the numerical, the total udl is converted into point load, acting on centre of UDL.



Uniformly Varying distributed Load:

A uniformly varying distributed load is on which is seperated/spread over a beam in a such manner that the rate of loading varies from point to point along the beam in which load is zero at one end and increases uniformly to other end.

- This type of load is known as triangular load

- For solving numerical, the total load is equal to of triangle and this total load is assumed to acting at the CG of triangle.

Concentrated Moment:

A concentrated moment is one which considered to act at a point.

Types of beam according to no. of reaction:

1. Statically determinant beam:

   If the total number of reaction in beam is less or equal than no. of useful static equlibrium then it is called statically determinant beam. Example, S.S.B., overhanging beam, Continuous beam.

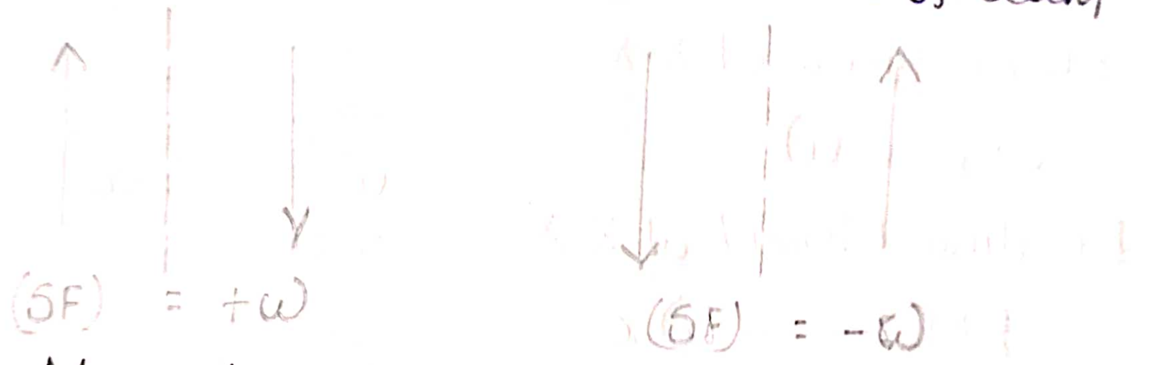
2. Statically indeterminat beam:

   If total no. of reaction in beam is more than no. of equilibrium equation, than it is called Statically indeterminant beam.

   Example; Fixed beam, Continuous Beam.

SHEAR FORCES

- Shear forces at any cross-section of beam is equal to algebric sum of vertical forces either on the LHS of cross-section of beam or RHS of cross-section of beam
- Shear force at any cross-section of beam is said to positive when it act on the upward direction on the LHS of cross-section of beam or when it is in the downward direction on the RHS of beam

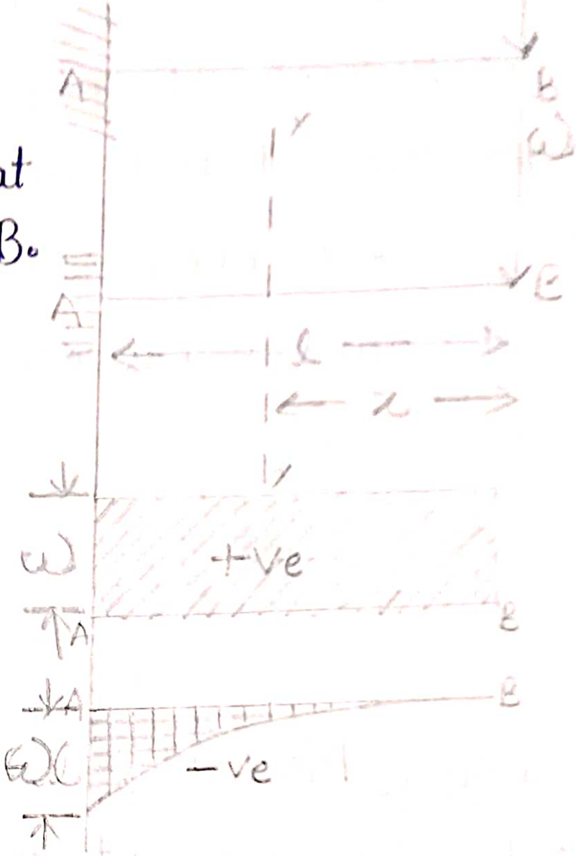


Bending Moment

- Bending moment acting at any cross-section is equal to algebric sum of couple or moment either on L.H.S. or on R.H.S. of the cross-section of the member.
- Bending moment at any cross-section of the member is said to be positive when it is acting in the clock-wise direction on the left of cross section of the member or when it is acting in anticlockwise on the right hand of the cross-section of the member.

Shear force & Bending moment diagram for a cantilever beam with a point load at the free end;

Consider a beam of length 'l' subjected to load 'W' at free end.



Now,
Let consider section XX at a distance of x from the B.

$BA = [x=0, x=l]$

Shear force at X-X

$SF_{xx} = W$

Bending Moment at X-X

$BM_{xx} = -Wx$

At $x=0$,

$SF = W$

$BM = 0$

At $x=l$,

$SF = W$

$BM = -Wl$.

Conclusion;

For point load;

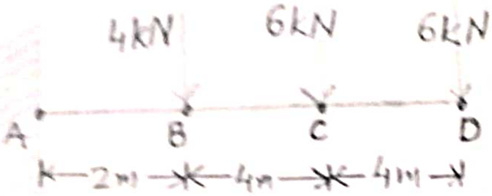
(i) Shear force diagram is constant line.

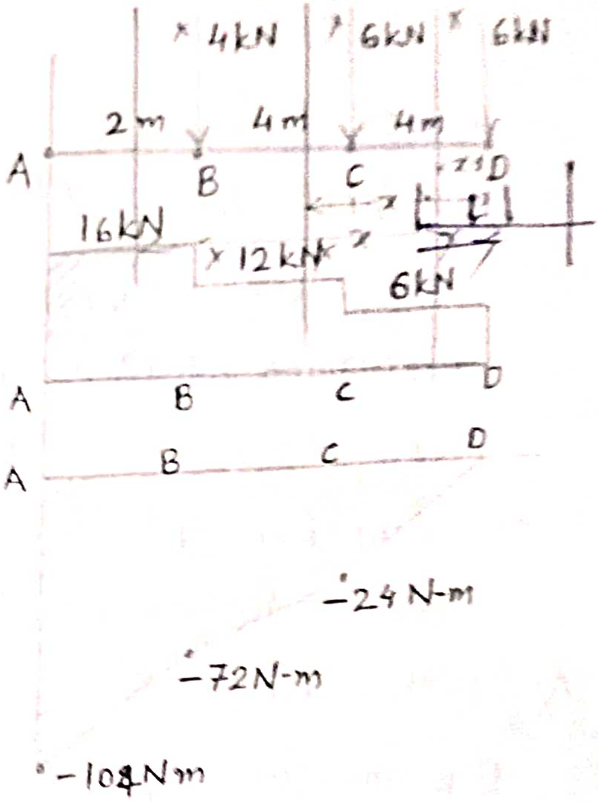
(ii) Bending moment is inclined straight line.

SHEAR FORCES AND Bending Moment diagram:

Calculation of Shear Force and Bending Moment in case of point load :—

Qn. 01. A cantilever is loaded as shown in fig. Draw shear force and bending moment.





At portion 'CD',

Shear force, V_{CD} = 6kN

Bending Moment, $M_{CD} = -6x$ N-m

∵ Shear force at 'D' = 6kN

Bending Moment at 'D' = 0

or shear force at 'C' = 6kN

Bending Moment at 'C' = -24 N-m

At portion 'BC',

Shear force, V_{BC} = 12kN

Bending Moment, $M_{BC} = -6x - 6(x-4)$

∵ Shear force at 'C' = 12kN

Bending Moment at 'C' = -24Nm

or Shear force at 'B' = 12kN

Bending moment 'B' = -72 N

At portion 'AB'

Shear force, V_{AB} = 16kN

Bending Moment $= -6x - 6(x-4) - 4(x-8)$

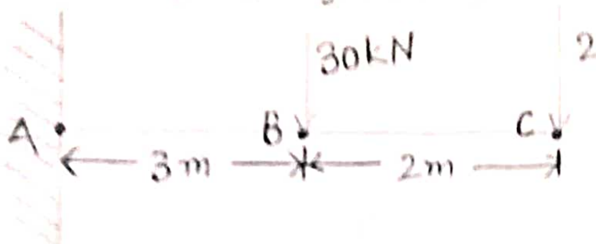
∵ Shear force at 'B' = 16kN

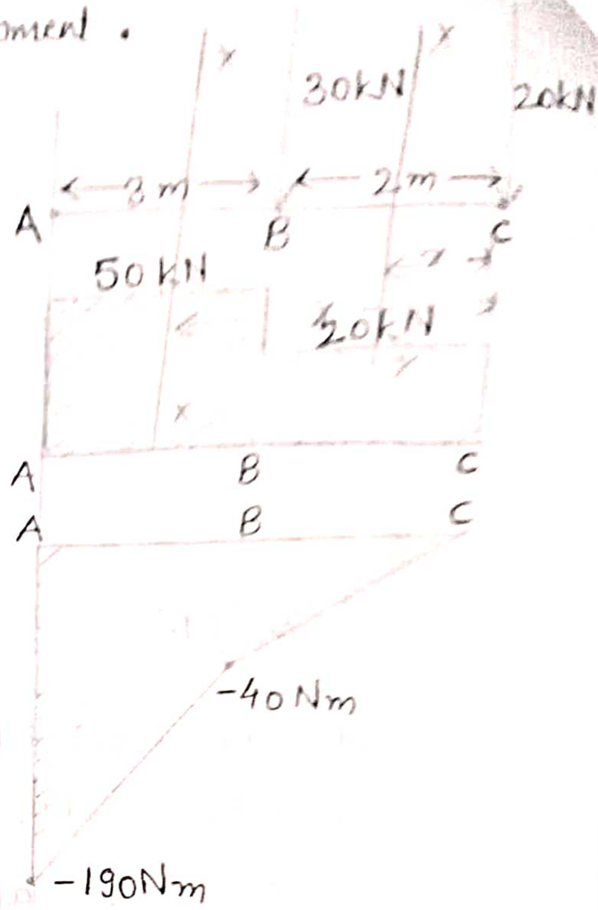
Bending Moment at 'B' = -72 N-m

or Shear force at 'A' = 16kN

Bending Moment at 'A' = -108 N-m

Qn 2. In the figure there is a centilever beam is loaded. Find shear force and bending moment.





At portion "BC";

Shear force, V_{BC} = 20kN

Bending Moment, $M_{BC} = -20 \, x \, x$ N-m

∴ Shear force at 'C' = 20kN

Bending Moment at 'C' = 0

or Shear force, 'B' = 20kN

Bending Moment at 'B' = -40N-m

At portion "AB"

Shear force, V_{AB} = 50kN

Bending Moment, $M_{AB} = -20x - 30(x-2)$ Nm

∴ Shear force at 'B' = 50kN

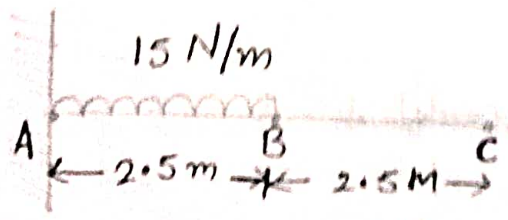
Bending Moment at 'B' = -40 N-m

or Shear force at 'A' = 50kN

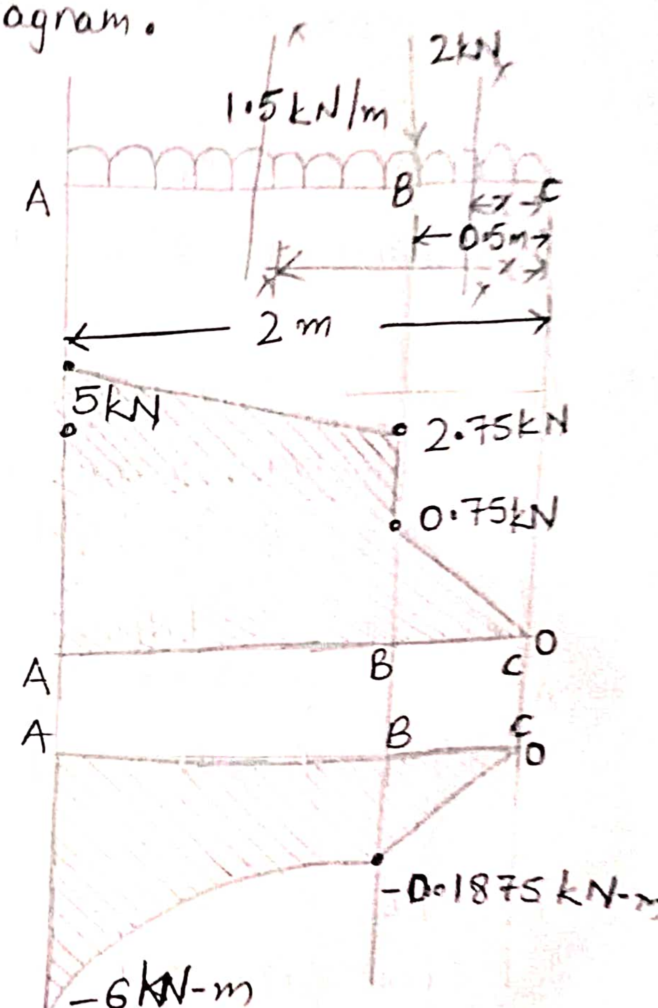
Bending Moment at 'A' = -190 N-m

Calculation of shear force and bending moment in case of Uniformly distribute Load :-

Qn. 3. Draw a shear force and bending moment diagram for a centilever beam as shown in fig.



Qn. 05 A cantilever of length 2 m carries a UDL of 1.5 kN/m run over the whole length and point load of 2 kN at a distance of 0.5 m from the free end. Draw Shear force & Bending Moment diagram.



Given,

total length of cantilever = 2 m

UDL = 1.5 kN/m

point load = 2 kN

for BC,

Shear force $V_{BC} = 1.5x$

Bending Moment $M_{BC} = -1.5 \times \frac{x^2}{2}$

$\therefore$ At point C, $x = 0$

$\because$ $V_C = 0$

$M_C = 0$

At point B $x = 0.5$

$V_B = 1.5 \times 0.5 = 0.75 kN$

$M_B = -1.5 \times \frac{0.5^2}{2} = -0.1875 KNm$

for AB,

Shear force $V_{AB} = 1.5x + 2$

Bending Moment $M_{AB} = -1.5\frac{x^2}{2} - 2(x - 0.5)$

$\therefore$ $V_B = 2.75 kN \ (x = 0.5)$

$M_B = -0.1875 kN$

$\therefore$ $V_A = 5 kN$

$M_A = -\frac{1.5 \times 2}{2} \ 2 \times 1.5 = -6 kN\text{-}m$

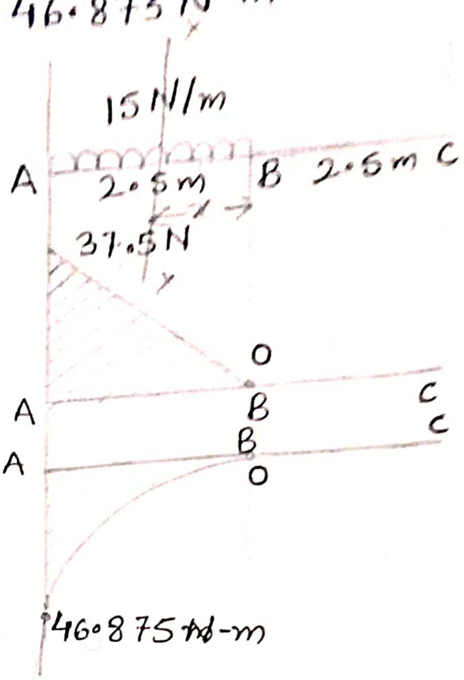
At 'AB',

Shear force $V_{AB} = 15 \times 2.5$
$= 37.5 N$

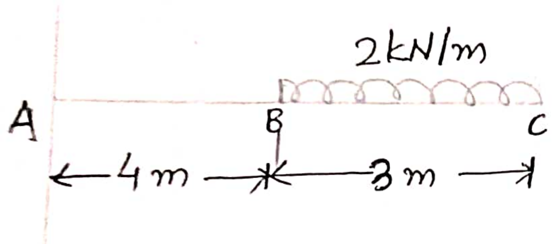
Bending moment $V_A = -\frac{15 \times 6.25}{2} = -46.875 N\text{-}m$

Shear force $V_B = 0$

Bending moment $V_B = 0$



Qn. 4. Draw a shear force and bending moment diagram, in case of centilever beam as shown in fig. below.



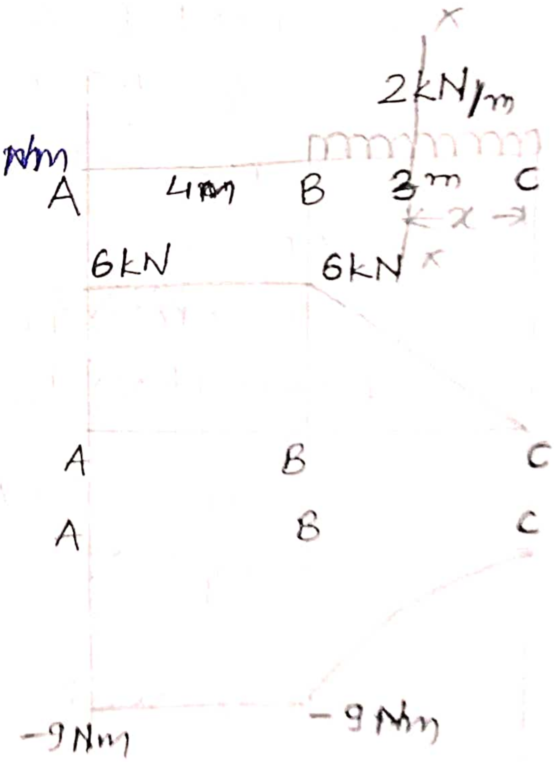
At BC,

Shear force at B $= 2 \times 3 = 6 kN$

Bending Moment at B $= -\frac{2 \times 9}{2} = -9 Nm$

Bending Moment at C $= 0$

Shear force at C $= 0$



At AB

Shear force at B $= 6 kN$

Bending Moment at B $= -9 Nm$

Shear force at A $= 6 kN$

Bending Moment At A $= -9 Nm$

Qn. 6

At AB,

Shear force $V_{AB} = 3kN$

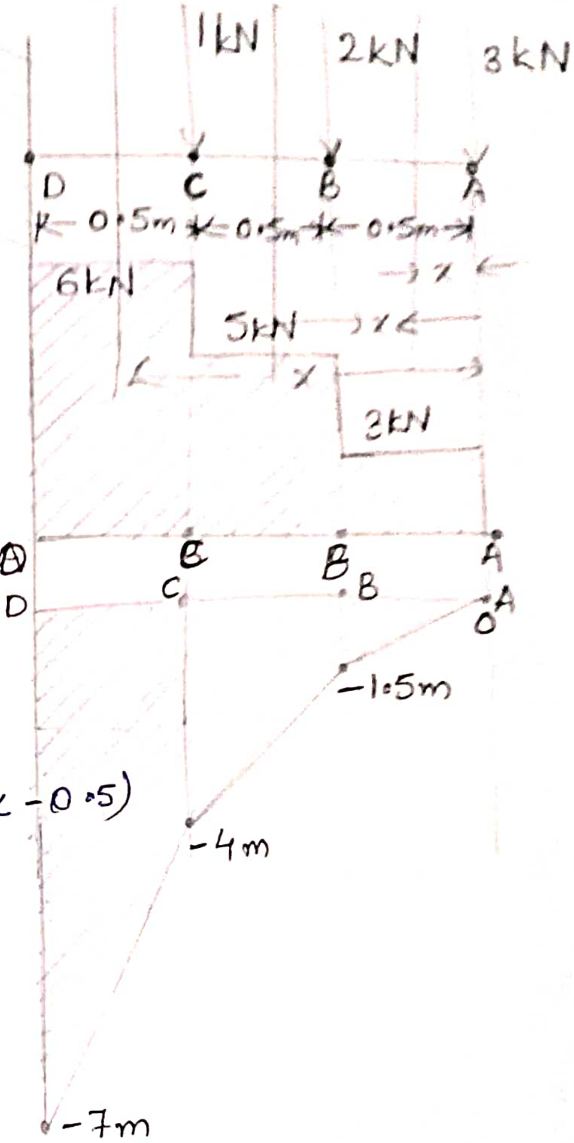
Bending Moment $M_{AB} = -3x$

point A

$V_A = 3KN$, $m_A = 0$

point B

$V_A = 3kN$, $m_B = -1.5m$



At BC,

Shear force. $V_{BC} = 5kN$

Bending Moment $M_{BC} = -3x - 2(x - 0.5)$

point B,

$V_B = 5kN$, $m_B = -1.5m$

point C,

$V_c = 5kN$, $m_c = -4m$

At CD,

Shear force, $V_{CD} = 6kN$

Bending Moment $m_{CD} = -3x - 2(x - 0.5) - 1(x-1)$

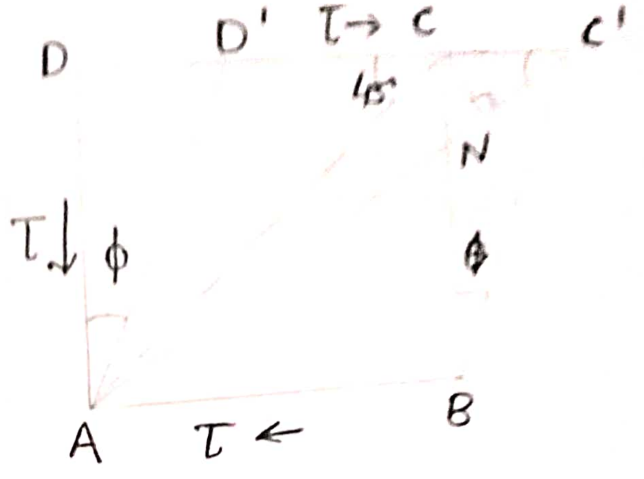
point C,

$V_c = 6kN$, $M_c = -4m$

point B,

$V_D = 6kN$, $M_D = -7m$

Relation between Elastic constant and modlus of rigidity:



$$\epsilon = \frac{FL - IC}{IC}$$

$$\epsilon = \frac{AC' - AC}{AC} = \frac{NC'}{AC}$$

$$\epsilon = \frac{CC' \cos 45^\circ}{\sqrt{2}\, BC}$$

$$\epsilon = \frac{1}{2} \times \frac{CC'}{BC}$$

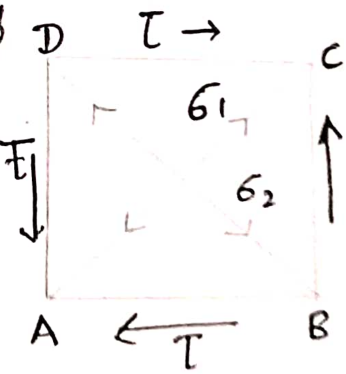
In $\Delta BCC'$

$$\tan\phi = \frac{CC'}{BC}$$

$$\epsilon = \frac{1}{2}\tan\phi$$

$\because$ ϕ is very small, then $\tan\phi = \phi$

$$\therefore \epsilon = \frac{1}{2}\phi$$



Strain diagonal 'AC'

$$\epsilon = \frac{\sigma_1}{E} - u\frac{\sigma_2}{E}$$

$$\epsilon = \frac{\tau}{E} - u\left(\frac{-\tau}{E}\right)$$

$$\epsilon = \frac{\tau}{E}(1+u)$$

$$\frac{1}{2}\phi = \frac{\tau}{E}(1+u)$$

$$\Rightarrow \tau \propto \phi \Rightarrow \tau = G\phi$$

$$\frac{\tau}{G} = \phi$$

$$\Rightarrow \frac{1}{2}\frac{\tau}{G} = \frac{\tau}{E}(1+u) \Rightarrow \boxed{E = 2G(1+u)}$$

UNIT-3 Theory of Simple Bending & Deflection of Beam

When some external load acts on a beam that shear force and bending moments set up at all section of beam.

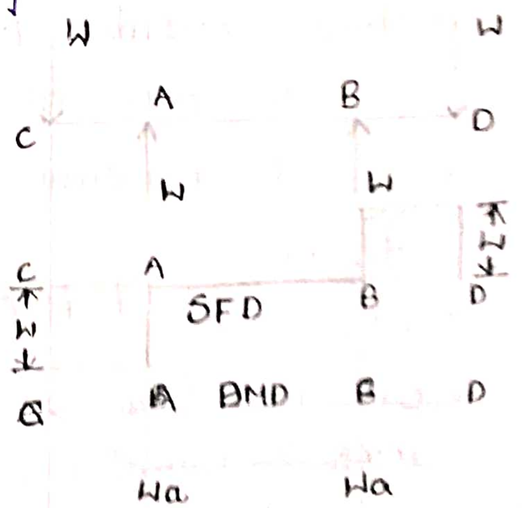
Due to this shear force and bending moment, the beam undergoes in certain deformation.

The material of the beam will offers stresses against this deformation.

The stress induced by shear force is called shear stress and the stress induced by bending moment is called bending stress.

Pure Bending : SIMPLE BENDING.

- If a length of a beam subjected to constant bending moment and no shear force then that length of beam is said to be in pure bending.
- The stress set-up in that length of beam due to bending moment is called bending stress.
- From diagram, it is clear that there is a shear force between A and B, but the bending moment between A & B is constant.
- The beam is subjected to constant bending moment between A & B.
- The condition of beam between A & B is known as "Pure Bending" or "Simple Bending"

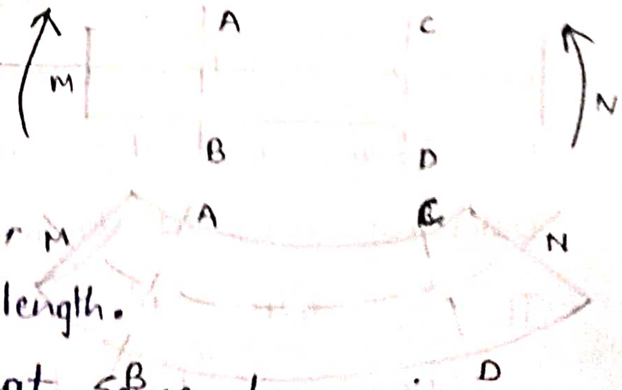


Assumption in Theory of Bending :

- The material of beam is homogenous and isotropic.
- The value of young's modulus of elasticity is the same in tension and compression.
- The transverse section which were plane before bending, remain plane after bending also.
- The material of beam obey "Hooke's Law".
- The radius of curvature is large compared with dimension of cross-section.
- The beam is initial straight and bend into circular arcs with common centre of curvature
- Each layer of beam in free to expand or contract, independently of the layer above or below it.

Theory of Simple Bending

- Consider a part of a beam subjected to simple bending. Again consider a small length δx of this part of beam.
- Now consider two section AB & CD which is normal to the axis of beam.
- Due to bending moment, the part of length δx will deform.
- From fig., it is clear that all the layer of beam which were originally of same length, do not remain of the same length.

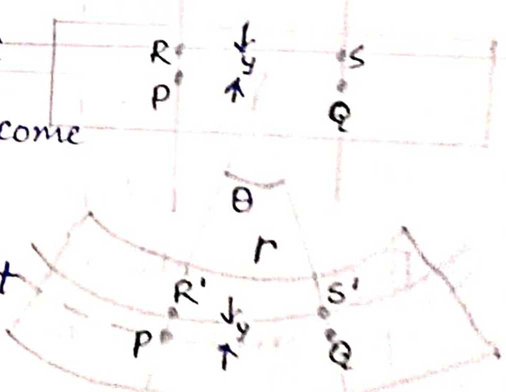


- From figure, it is clear that some layer is shortened while some elongaton.

- The top layer is shortended maximum, this mean compressive stress is max^m at the top.
- The bottom layer elongated maximum, this mean that tensile stress is maximum at the bottom.
- At ~~the~~ layer between top & bottom layer, there will be a layer which is nither shortended nor elongated. This layer is called Neutral layer. At the neutral layer bending stress is zero.
- Above Neutral layer, there will compressive stress & below Neutral layer there will be tensile stress.

Bending Equation or Flexural Equation :

Consider a small length dx of a beam subjected to bending moment. Due to this moment, let this small length of beam bend into an arc of circle with 'O' as centre.



Consider a layer RS at a distance of y from PQ which after bending become R'S'.

Let PQ is substendo an angle α at the centre of curvature.

$$P'Q' = \alpha r \quad \left[\theta = \frac{l}{R}\right]$$

$$r's' = \alpha(r-y)$$

Strain in rs, $\varepsilon = \dfrac{rs - r's'}{rs} = \dfrac{p'q' - r's'}{rs}$

$$= \frac{r\alpha - (r-y)\alpha}{r\alpha} = \frac{ry}{r\alpha} = \frac{y}{\alpha}$$

$$\therefore \boxed{\varepsilon = \frac{y}{\alpha}}$$

Note :

If stress in RS = σ_b and young modulus = E,

$\therefore$ strain = $\frac{\sigma_b}{E}$

$$\varepsilon = \frac{\sigma_b}{E}$$

$$\frac{y}{R} = \frac{\sigma_b}{E}$$

$$\boxed{\frac{\sigma_b}{y} = \frac{E}{R}}$$

Now, consider transverse section of beam, let a strip of dA lies at at a distance of y from Neutral axis.

Normal force of Area dA . df

$$df = \sigma_b \, dA$$

Moment of df about Neutral axis,

$$dM_R = dF \times y$$

$$dM_R = \sigma_b \, dA \times y$$

$$dM_R = \frac{Ey}{R} \, dA \, y$$

$$dM_R = \frac{E}{R} y^2 \, dA$$

This is the resisting moment of the material caused by stress produced and resisting moment.

$$\int dM_R = \int \frac{E}{R} y^2 \, dA$$

$$M_R = \frac{E}{R} \int y^2 \, dA$$

$$M_r = \frac{E}{R} . I$$

$$\boxed{\frac{M_R}{I} = \frac{E}{R}} \quad \text{And} \quad \boxed{\frac{M_R}{I} = \frac{E}{R} = \frac{\sigma_B}{y}}$$

Note : • For safe condition $M \leq M_R$

• For static equilibrium, $M = M_R$

• The beam having higher M_R is best beam.

Moment of Resistance :

• Due to pure bending, the layer above the neutral axis are subjected to compressive stresses where as the layer below the neutral axis are subjected to tensile stresses.

• Due to these stresses the force will be acting on the layers.

• These forces will have moment about "Neutral axis".

• The total moment of these forces about the Neutral axis for a section is known as moment of resistance of that section.

Analysis of bending equation :

$$\boxed{\frac{M_R}{I} = \frac{\sigma_b}{y} = \frac{E}{R}}$$

$$A = B = C$$

Where,

M_R - Bending moment

I - Moment of inertia

σ_b - Bending stress

y - Distance from Neutral axis

E - Young Modulus

R - Radius of curvature

Case I : $A = B$

$$\frac{M_R}{I} = \frac{\sigma_b}{y}$$

$$\sigma_b = \frac{M_R\, y}{I}$$

This equation is used to determine σ_b develop at the fibre loaded at a distance of y from Neutral Axis when bending acting at the cross-section is known.

From eq^n, $6_b = y$

$$(6_b)_{max} = \frac{M\, y_{max}}{I}$$

$$\boxed{(6_b)_{max.} = \frac{M}{\frac{I}{y_{max.}}} = \frac{M}{Z_{NA}}}$$

Where Z_{NA} = Section modulus of x-s/c of a beam about its neutral axis

$$\boxed{Z_{NA} = \frac{I}{y_{max}}}$$

Note : (i) $(6_b)_{max} \propto \frac{1}{Z_{NA}}$

$Z_{NA} \uparrow \rightarrow 6_b\ max \downarrow \rightarrow$ Changes of failure is less.

(ii) $6_b \propto y$, $(6_b)_{max} \propto y_{max}$

$$\frac{6_b}{(6_b)_{max}} \propto \frac{y}{y_{max}} \Rightarrow \boxed{6_b = (6_b)_{max} \times \frac{y}{y_{max}}}$$

(iii) $\frac{6_b\ top}{6(bottom)} = \frac{y_{top}}{y_{bottom}}$

(iv) $\frac{(6_b)\ top}{(6_b)\ bottom} = -1$ []

$= -2$ []

CASE II : B = C

$$\frac{6_b}{y} = \frac{E}{R} \Rightarrow 6_b = y\frac{E}{R} \Rightarrow \boxed{(6_b)_{max} = \frac{E}{R} y_{max}}$$

The equation is used to determine 6_b develop on a cross-section of beam when radius of curvature 'R' of Neutral fibre is known.

Note : $(6_b)_{max} \propto E y_{max}$

$E \uparrow$ & $y_{max} \downarrow \Rightarrow (6_b)_{max} \downarrow$ = Change in failure.

CASE III : $A = C$

$$\frac{M_R}{I} = \frac{E}{R}$$

$$\boxed{EI = M_R \times R}$$

For $R = 1$, $\boxed{M = EI}$.

$EI \uparrow \rightarrow M_R \uparrow$

→ Deflection of slope ↓

→ Changes in bending failure ↓.

Section Modulus :

The ratio of moment of inertia of intertia of a section about the neutral axis to the distance of outermost layer from neutral axis.

- It is denoted by 'Z'.
- $\boxed{Z = \frac{I}{y_{max}}}$

Section Modulus for various shapes or beam section ;

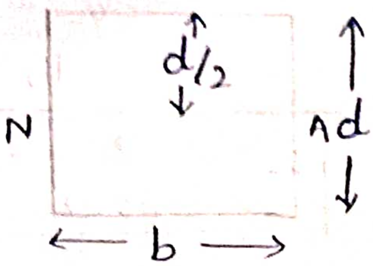
1. Rectangular Section ;

Moment of Inertia of ractangular section about an axis through its C.G. or N.A. is given by

$$I_{NA} = \frac{bd^3}{12}$$

Distace of outer mast layer from NA

$$y_{max} = d/2$$



$$Z = \frac{I}{y_{max}} = \frac{bd^{\cancel{3}2}}{6\,\cancel{12} \times \cancel{d}} = \frac{bd^2}{6}$$

$$Z = \frac{bd^2}{6}$$

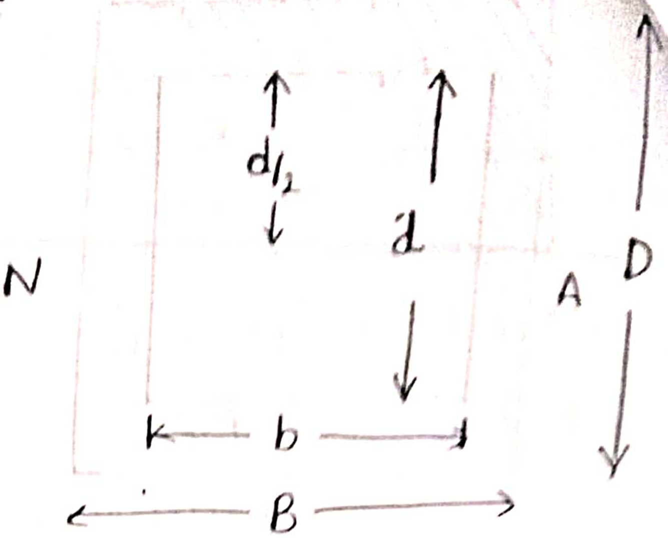
2. Hollow ractangular section:

$$I = \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$y_{max} = \frac{D}{2}$$

$$Z = \frac{\frac{1}{12}\left[BD^3 - bd^3\right]}{\frac{D}{2}}$$

$$\boxed{Z = \frac{1}{6D}\left[BD^3 - bd^3\right]}$$



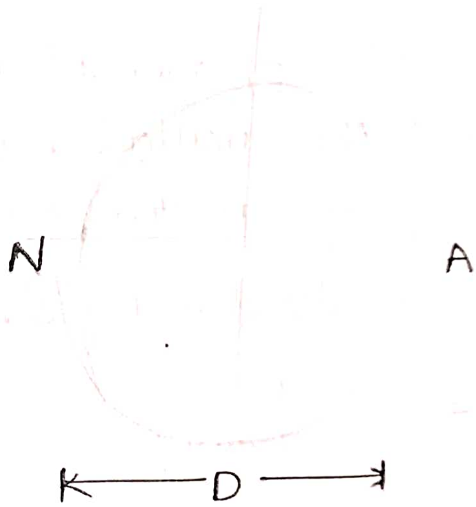
3. Circular Section:

$$I = \frac{\pi}{64} D^4$$

$$y_{max} = D/2$$

$$Z = \frac{I}{y_{max}} = \frac{\frac{\pi}{64} D^4}{D/2}$$

$$\boxed{Z = \frac{\pi}{32} d^3}$$



4. Hollow circular section:

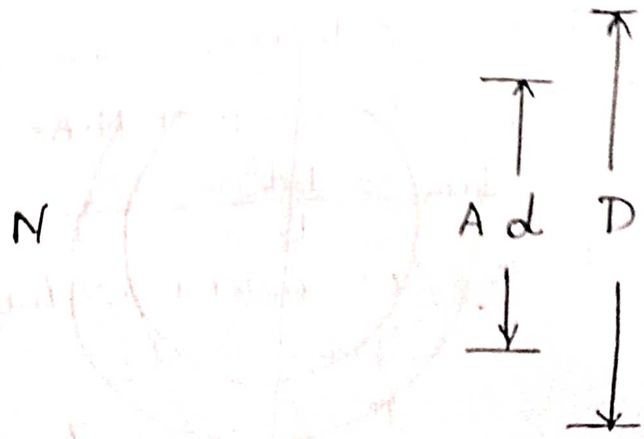
$$I = \frac{\pi}{64}(D^4 - d^4)$$

$$y_{max} = \frac{D}{2}$$

$$Z = \frac{I}{y_{max}}$$

$$Z = \frac{\frac{\pi}{64}(D^4 - d^4)}{D/2}$$

$$\boxed{Z = \frac{\pi}{32D}(D^4 - d^4)}$$



Deflection.

Bending of beam: The deviation of axis of beam due to load acting on it is called bending of beam.

Buckling of column: The deviation of column due to load acting on it is known as buckling of column.

Beflection of beam:

The leanear deviation of axis of beam while it is bending is known as deflection of beam.

It is denoted by 'y'.

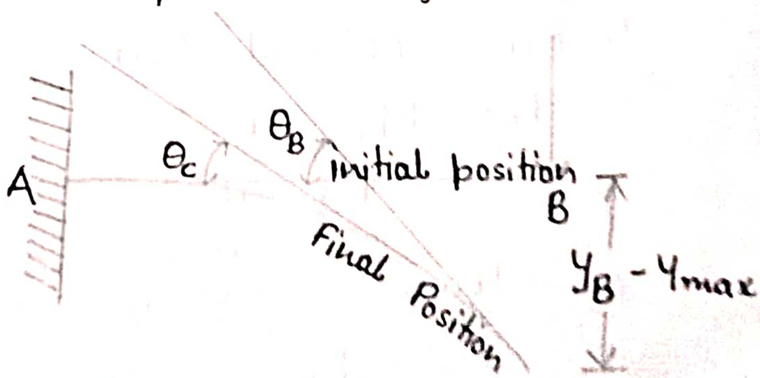
Slope of beam:

The angular deviation of beam axis of beam while it is bending is called slope.

It is denoted by $\frac{dy}{dx}$ or θ.

- In this chapter, slope and deflection due to shearforce is neglected because slope and deflection due to this force is very small as compared to slope and deflection due to bending moment.

For Cantilever beam:



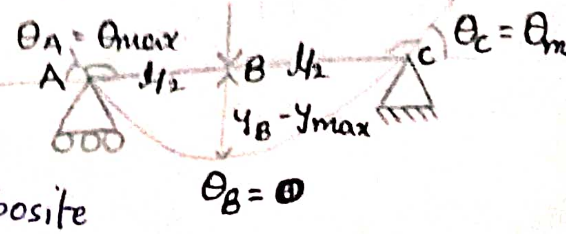
At Free End;

$\theta = \theta_{max}$ & $y = y_{max}$

At Fixed End;

$\theta = 0$ & $y = 0$.

For Simple supported beam with symmetric load.



At mid point, $\theta = 0$ & $y = y_{max}$

At support $\theta = \theta_{max}$ & $y = 0$

Slope are equal but both are opposite

Deflection.

For simple supported beam with unsymmetric load;

Deflection is maximum in the region of point of application of load & mid span.

Region of y-max and slope

Slop is maximum at the supported which is nearer to applied load.

Slop is zero where deflection is maximum.

Calculation of deflection by double integration method;

$$\boxed{M_{x-x} = [EI]_{xx} \left[\frac{d^2y}{dx^2}\right]_{xx}}$$

Note, $\frac{dy}{dx} = \tan\theta$

For small angle

$\tan\theta \cong \theta$

$$\therefore \boxed{\theta = \frac{dy}{dx}}$$

M_{xx} - Bending moment at section x-x

$\frac{dy_{xx}}{dx}$ - Slope at section x-x

Y_{xx} - Deflection at section x-x

E - Modulus of Elasticity

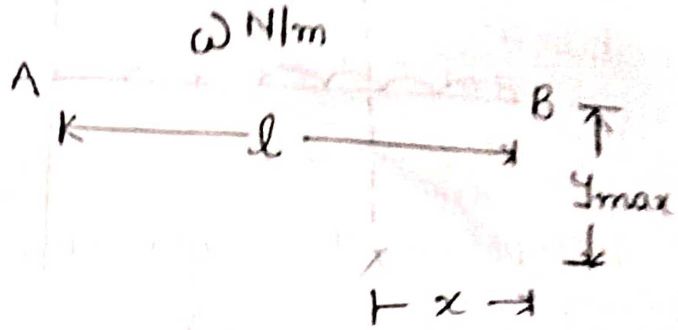
I - Moment of inertia.

Steps of calculation;

1. First calculate M_{xx}.
2. Put M_{x-x} in
   $$EI \frac{d^2y}{dx^2} = M_{xx}$$
3. Integrate this equation
   $$EI \frac{dy}{dx} = \int M_{xx}\,dx + C_1$$
   It is determined by boundary condition of Slope.
4. Again intigrate this equation.
   $$EI\,Y_{xx} = \int\left[\int M_{xx}\,dx\right]dx + \int C_1\,dx + C_2$$
   It is determined by boundary condition of deflection.

Deflection of cantilever beam of length 'L' carrying UDL ω per unit length run over whole length;

Let a section X-X at a distance of 'x' from 'B'.



$$M_{x-x} = -\frac{\omega x^2}{2}$$

$$EI\frac{d^2y}{dx^2} = M_{xx}$$

$$\Rightarrow EI\frac{d^2y}{dx^2} = -\frac{\omega x^2}{2}$$

$$\Rightarrow \int EI\frac{d^2y}{dx^2} = \int -\frac{\omega x^2}{2}$$

$$\Rightarrow EI\frac{dy}{dx} = -\frac{\omega x^3}{6} + C_1$$

At, $x = l$, $\frac{dy}{dx} = 0$

$$\Rightarrow EI \times 0 = -\frac{\omega l^3}{6} + C_1$$

$$\Rightarrow C_1 = \frac{\omega l^3}{6}$$

$$\therefore \boxed{EI\frac{dy}{dx} = -\frac{\omega x^3}{6} + \frac{\omega l^3}{6}}$$ General Equation of slope.

At $x = 0$, $\frac{dy}{dx} = \theta_{max}$

$$EI\,\theta_{max} = -\frac{\omega \cancel{0}^2}{6} + \frac{\omega l^3}{6}$$

$$\boxed{\theta_{max} = \frac{\omega l^3}{6EI}}$$

Again integrating,

$$EI\,y = \frac{-\omega x^4}{24} + \frac{\omega l^3}{6}x + C_2$$

At $x = l$, $y = 0$

$$0 = \frac{-\omega l^4}{24} + \frac{\omega l^4}{6} + C_2$$

$$C_2 = \frac{-4\omega l^4 + \omega l^4}{24} = -\frac{\omega l^4}{8}$$

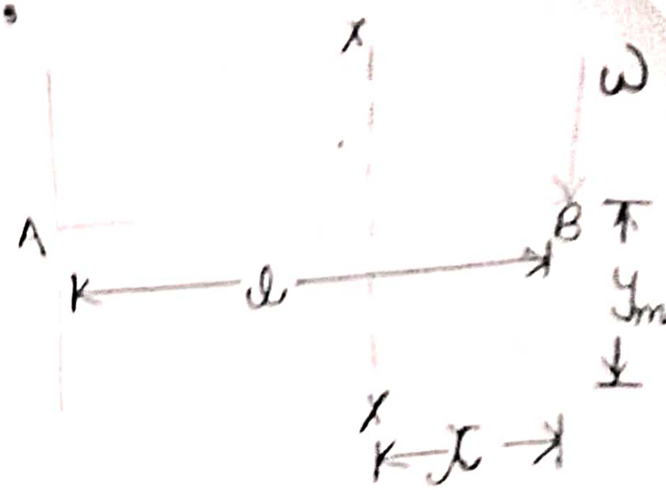
$$\therefore \boxed{EI\,y = \frac{-\omega x^4}{24} + \frac{\omega l^3 x}{6} - \frac{\omega l^4}{8}}$$ General Eqn of Deflection.

At $x = 0$, $y = y_{max}$

$$EI\,y_{max} = -0 + 0 - \frac{\omega l^4}{8}$$

$$\boxed{y_{max} = -\frac{\omega l^4}{8EI}}$$

Deflection of centilever beam of length 'l' carries point at a distance of 'x' from free end:



$$M_{x-x} = -\omega x$$

$$EI\frac{d^2y}{dx^2} = M_{x-x}$$

$$EI\frac{d^2y}{dx^2} = -\omega x$$

Integrating this equation,

$$EI\frac{dy}{dx} = -\frac{\omega x^2}{2} + C_1$$

At $x = l$, $\frac{dy}{dx} = 0$

$$0 = -\frac{\omega l^2}{2} + C_1$$

$$C_1 = \frac{\omega l^2}{2}$$

$$\therefore \boxed{EI\frac{dy}{dx} = -\frac{\omega x^2}{2} + \frac{\omega l^2}{2}}$$ General eqn of slope.

At $x=0$, $\frac{dy}{dx} = \theta_{max}$

$$EI\,\theta_{max} = 0 + \frac{\omega l^2}{2}$$

$$\boxed{\theta_{max} = \frac{\omega l^2}{2EI}}$$

At $x = 0$, $y = y_{max}$

$$\boxed{y_{max} = -\frac{\omega l^3}{3EI}}$$

Again by integrationg,

$$EI\,y = -\frac{\omega x^3}{6} + \frac{\omega l^2 x}{2} + C_2$$

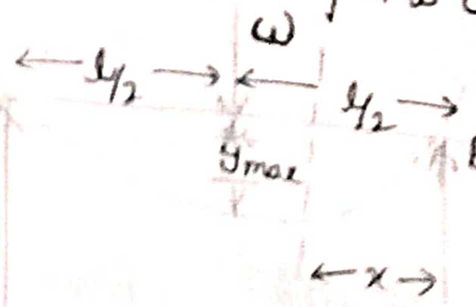
At $x = l$, $y = 0$

$$0 = -\frac{\omega l^3}{6} + \frac{\omega l^3}{2} + C_2$$

$$C_2 = \frac{\omega l^3 - 3\omega l^3}{6} = -\frac{\omega l^3}{3}$$

$$\therefore \boxed{EI\,y = -\frac{\omega x^3}{6} + \frac{\omega l^2 x}{2} - \frac{\omega l^3}{3}}$$ General equation of Deflection

Deflection of simple supported beam of length 'l' carrying point load;



Consider section X-X at distance of x from B.

$$M_{x-x} = \frac{\omega}{2} x$$

$$EI \frac{d^2y}{dx^2} = M_{xx}$$

$$EI \frac{d^2y}{dx^2} = \frac{\omega}{2} x$$

Intigrating the above equation,

$$EI \frac{dy}{dx} = \frac{\omega x^2}{4} + C_1$$

At $x = l/2$, $\frac{dy}{dx} = 0$

$$0 = \frac{\omega l^2}{16} + C_1$$

$$C_1 = -\frac{\omega l^2}{16}$$

$$\therefore \boxed{EI \frac{d^2y}{dx^2} = \frac{\omega}{4} x^2 - \frac{\omega l^2}{16}}$$ General equation of slope.

At, $x = 0$, $\frac{dy}{dx} = \theta max$

$$\therefore \boxed{\theta_{max} = -\frac{\omega l^2}{16EI}}$$

Again integrating,

$$EIy = \frac{\omega}{12} x^3 - \frac{\omega l^2 x}{16} + C_2$$

At $x = 0$, $y = 0$

$$0 = 0 - 0 + C_2$$

$$\therefore C_2 = 0$$

$$\therefore \boxed{EIy = \frac{\omega}{12} x^3 - \frac{\omega l^2 x}{16}}$$ General Equation of Deflection.

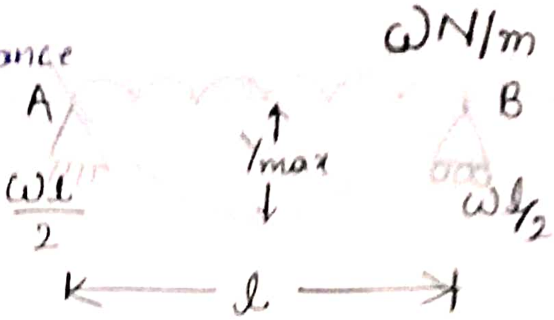
At $x = l/2$, $y = max$

$$\therefore EI y_{max} = \frac{\omega l^3}{96} - \frac{\omega l^3}{32}$$

$$EI y_{max} = -\frac{\omega l^3}{48}$$

$$\Rightarrow \boxed{y_{max} = -\frac{\omega l^3}{48EI}}$$

Deflection of simple supported beam of length 'l' carrying UDL over the entire length;



Consider the section X-X at distance of x from B,

$$M_{x-x} = \frac{\omega l x}{2} - \frac{\omega x^2}{2}$$

$$EI\frac{d^2y}{dx^2} = \frac{\omega l x}{2} - \frac{\omega x^2}{2}$$

Integrating the above eqn,

$$EI\frac{dy}{dx} = \frac{\omega l x^2}{4} - \frac{\omega x^3}{6} + C_1$$

At $x = l/2$, $dy/dx = 0$

$$0 = \frac{\omega l^3}{16} - \frac{\omega l^3}{48} + C_1$$

$$C_1 = \frac{-3l^3\omega + l^3\omega}{48} = -\frac{\omega l^3}{24}$$

$$\therefore \boxed{EI\frac{dy}{dx} = \frac{\omega l x^2}{4} - \frac{\omega x^3}{6} - \frac{\omega l^3}{24}}$$ General equation of slope.

At $x = 0$, $\frac{dy}{dx} = \theta_{max}$

$$\boxed{\theta_{max} = -\frac{\omega l^3}{24EI}}$$

Again integrating,

$$EIY = \frac{\omega l x^3}{12} - \frac{\omega x^4}{24} - \frac{\omega l^3 x}{24} + C_2$$

At $x = 0$, $y = 0$

$$0 = 0 - 0 - 0 + C_2$$

$$C_2 = 0$$

$$\therefore \boxed{EIY = \frac{\omega l x^3}{12} - \frac{\omega x^4}{24} - \frac{\omega l^3 x}{24}}$$ General equation of deflection.

At $x = l/2$, $y = y_{max}$

$$EI\,y_{max} = \frac{\omega l^4}{96} - \frac{\omega l^4}{384} - \frac{\omega l^4}{48}$$

$$\boxed{y_{max} = \frac{5\omega l^4}{384EI}}$$

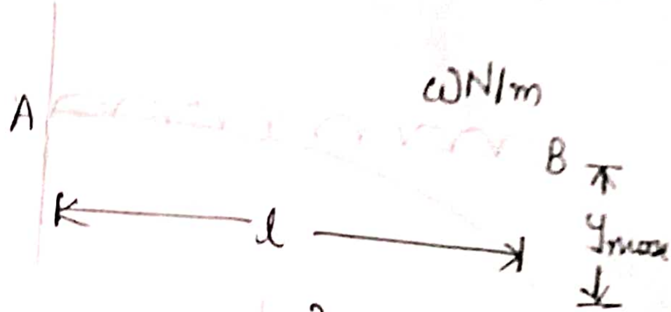
Note:

(I) $\theta_A = 0$, $\theta_{B_{max}} = \dfrac{\omega l^2}{2EI}$

$y_A = 0$, $y_{B_{max}} = \dfrac{\omega l^3}{3EI}$

(II) $\theta_A = 0$ $\quad \theta_{B_{max}} = \dfrac{\omega l^3}{6EI}$

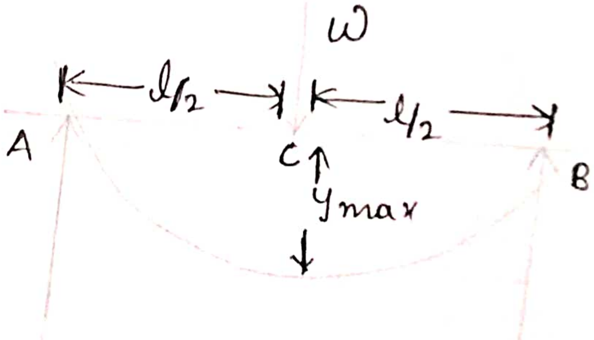
$y_A = 0$ $\quad y_{B_{max}} = \dfrac{\omega l^4}{8EI}$



(III) $(\theta_A)_{max} = (\theta_B)_{max} = \dfrac{\omega l^2}{16EI}$

$y_A = y_B = 0$

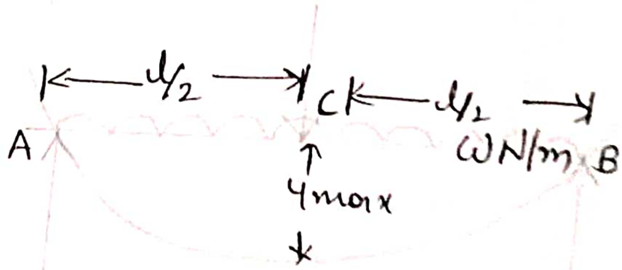
$\theta_c = 0$, $y_{c\,max} = \dfrac{\omega l^3}{48EI}$



(IV) $(\theta_A)_{max} = (\theta_B)_{max} = \dfrac{\omega l^3}{24EI}$

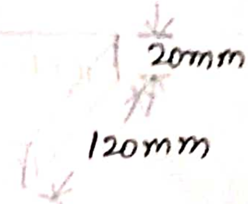
$y_A = y_B = 0$

$\theta_c = 0$, $(y_c)_{max} = \dfrac{5\,\omega l^4}{384EI}$



Question

Qn. 1. A steel plate width 120 mm and thickness 20mm is bend into a circular arc of radius 10 m. Determine the maximum stress induced and bending moment which will produce the maximum stress. Take $E = 2\times10^5\ N/mm^2$.



Given,

Width, $b = 120\,mm$

thickness, $t = 20\,mm$

Radius, $R = 10\,m = 10000\,mm$, $E = 2\times10^5\ N/mm^2$

$\because\ t = 20\ mm$

$\therefore\ y_{max} = 10\ mm$

then, max^m bending stress $\sigma_{b\,max} = \frac{E}{R}\times y_{max}$

$$= \frac{2\times10^5}{10000}\times 10$$

$$= 200\ N/mm^2.$$

Then,

$$I_{NA} = \frac{b\,d^3}{12} = \frac{\cancel{120}^{10}\times 20^3}{\cancel{12}}$$

$$= 80000\ mm^4$$

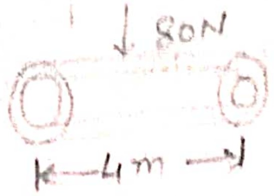
$$\therefore\ \frac{M}{I_{NA}} = \frac{E}{R}$$

$$M = \frac{E\times I_{NA}}{R} = \frac{2\times10^5\times 80000}{10000} = 1600000\ Nmm$$

$$\Rightarrow 16\,000\ Nm$$

$$\Rightarrow 1.6\ KNm$$

Qn. Calculate maximum stress induced in a cast iron pipe of external diameter 40mm internal diameter 20 mm and length 4 m when pipe is supported at its and carries a point load of 80N at its centre.



Given, External Diameter = 40mm

Internal Diameter = 20mm

length = 4 m = 40000 mm

[illegible] = 80N

Maximum bending moment

$$M = \frac{Wl}{4}$$

$$= \frac{80 \times 4}{4} = 80\,Nm = 80 \times 1000\,Nmm$$

then, Moment of inertia,

$$I_{NA} = \frac{\pi}{64} \times (D^4 - d^4)$$

$$= \frac{\pi}{64 \times 10} \times (40^4 - 20^4)$$

$$= \frac{\pi}{64 \times 10} \times 2400000$$

$$= \frac{\pi \times 2400000}{640} = 5890.48$$

$$= \pi \times 37500 = 117809.7\,mm^4$$

$$\because \frac{M}{I_{NA}} = \frac{\sigma_b}{y}$$

$$\Rightarrow \sigma_b = \frac{My}{I_{NA}} = \frac{80 \times 20 \times 1000}{117809.72}$$

$$= 13.58\,N/mm^2.$$

Qn. A ractangular beam 200mm deep and 300mm wide is simply supported over a span of 8m what U.D.L per meter the beam may carry if bending stress is not to axceed 120 N/mm².

Given,

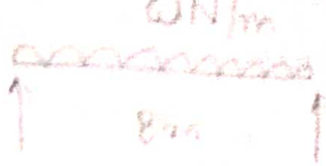
Depth = 200mm

Width = 300 mm

length = 8m = 8000mm

Bending Stress = 120 N/mm²

Let, ω N/m UDL acting on a beam,



Maximum bending Moment,

$$M_{max} = \frac{\omega l^2}{8} = \omega 8 \text{ Nm}, \quad y_{max} = \frac{200}{2} = 100\text{mm}$$

$$I_{NA} = \frac{bd^3}{12} = \frac{\cancel{300} \times 200^3}{\cancel{12}} = 50\times10^{4}$$

$$= 20\times10^7$$

$$\because \frac{M}{I_{NA}} = \frac{\sigma_b}{y}$$

$$= \frac{8000\,\omega}{20\times10^7} = \frac{120}{100}$$

$$\Rightarrow \omega = \frac{\cancel{120}^{60} \times \cancel{20}^{5} \times 10^{7}}{1000 \times \cancel{100} \times \cancel{8}} = 30000 \text{ N/m}$$

Qn. A square beam 20×20 mm in section and 2m long is supported at the end of the beam fail when a point load of 400N is applied at the centre of beam. What U.DL per meter length will break a cantilever of same material 40 mm wide 60mm deep and 3 m long.

Given, Depth of beam = 20 mm , y = 10 mm
Width of beam = 20mm
length of beam = 2 m = 2000 mm
Point load = 400N

$\because$ Max^m bending moment, $\frac{W.l}{4} = \frac{400 \times 2}{4} = 200$ Nm
$= 200\times10^3$ Nmm

$$I_{NA} = \frac{20\times20^3}{12} = \frac{20^4}{12}$$

$$\sigma_b = \frac{M\times y}{I_{NA}} = \frac{200\times10^3 \times 10 \times 12}{20^4} = 150 \text{ N/mm}^2$$

(ii)

then,

$b = 40mm$

$d = 60mm$, $y = 30mm$ & $l = 3m$

$I_{NA} = \frac{\overset{10}{\cancel{40}} \times 60^3}{\cancel{12}_3} \; \frac{10 \times \overset{72}{\cancel{216}} \times 10^3}{\cancel{3}} = 72 \times 10^4 \; mm^4$

Let w be the stress which work on the UDL,

∵ Max^m bending moment,

$M_{max} = \frac{wl^2}{\cancel{2}} = \frac{w \cancel{0} 9\cancel{00} \times 10^4}{8}$

∵ $\frac{M}{I_{NA}} = \frac{\sigma_b}{y}$

$\Rightarrow w \frac{9}{\cancel{2}} 10^3 = \frac{\overset{5}{\cancel{150}} \times 72 \times 10^4}{\cancel{30}}$

$= w = \frac{8 \times 5 \times \overset{36}{\cancel{72}} \times 10^4}{\cancel{2} \times \cancel{10^3}} = \cancel{0}8\,00 N/mm.$

Qn. A beam 6 m long, simply supported at its ends is carrying a point load at its centre carrying a point load of 50kN at its centre. The moment of inertia of beam is equal to $78 \times 10^6 \; mm^4$. If E of the material of beam is $2.1 \times 10^5 \; N/mm^2$ calculate

(i) Deflection at centre of beam

(ii) Slope of support.

Given, length of beam $= 6m = 6 \times 10^3$ Mm

point load $= 50$ kN

$I_{NA} = 78 \times 10^6 \; mm^4$

$E = 2.1 \times 10^5 \; N/mm^2$

(i) $y = \frac{Wl^3}{48 E \times I} = \frac{5\cancel{0000} \times 6^3 \times 10^{9}}{48 \times 78 \times 10^6 \times 2.1 \times 10^5} = \frac{216 \times 5000}{78 \times 48 \times 21}$

$= 13.736$ mm

ii) Slope at support

$$\theta_B = \frac{wl^2}{16\,EI} = \frac{5\cancel{0000} \times 6^2 \times 10^6}{16 \times 211 \times 10^7 \times 78 \times 10^6} = \frac{180}{16 \times 211 \times 78}$$

$$= 0.06868.$$

$\because \theta_B = 0.06868^\circ$ or 3.935 rad

$\theta_A = -0.06868^\circ$ or -3.935 rad.

Qn. A beam 3m long simply supported at its ends is carrying point load N at centre, if slop at the end of beam should not exceed 1°. find the deflection.

given, length l, $= 3m = 3000$ mm

Slope at end, $\theta_B = 1^\circ = \frac{\pi}{180}$ rad

Let y = deflectio at mid-point of beam

$$y = \frac{wl^3}{48EI}$$

$$= \frac{wl^2}{16\,EI} \times \frac{l}{3}$$

$$y = \theta_B \frac{l}{3}$$

$$= \frac{\pi}{18\cancel{0}} \times \frac{\cancel{3}\cancel{000}\,100\cancel{0}}{\cancel{3}}$$

$$y = 17.45 \text{ mm}$$

Qn. A beam of length 5m and of uniform ractangular section is simply support at its ends. It carries a UDL of 9 kN/m runs over the entire length. Calculate the width and depth beam if permissible bending stress is 7 N/mm2 and central deflection is not to be exceed 1cm. E = 1×10^4 N/mm2.

Given, length of beam $l = 5m = 5000mm$

UDL, $w = 9 kN/m$

bending stress $= \sigma_B = 7 N/mm^2$

Central deflection, $y = 1 cm = 10 mm$

$E = 1 \times 10^4 \; N/mm^2$

$b =$ Width of beam

$d =$ depth of beam

$\therefore I = \frac{bd^3}{12}$

$\because y = \frac{5 w l^4}{384 EI}$

$$10 = \frac{5 \times \frac{90000}{\cancel{1000000}} \times 5^4 \times 10^{7} \times 12}{384 \times \cancel{10^4}\, bd^3} = \frac{60 \times 625 \times 10^7}{384 \times bd^3}$$

$$bd^3 = \frac{3750 \times 10^7}{384} = 878.906 \times 10^7 \, mm^4$$

$\because \sigma_{B\,max} = 7 N/mm^2$

$$\because M = \frac{wl^2}{8} = \frac{9 \times 5 \times 5000}{8} = 28125000 \, Nmm$$

$\because \frac{M}{I} = \frac{\sigma_b}{y}$

$$\frac{28123000}{\frac{bd^3}{12}} = \frac{7}{d/2}$$

$bd^2 = 24107142.85 \, mm^3$

$\because bd^2 \times d = 838.906 \times 10^7$

$$d = \frac{838.907 \times 10^7}{24107142.85} = 364.5 \, mm$$

$\therefore b = 181.36 \, mm$

... a point load of 50kN at a distance of 2m from fixed end. If $I = 10^8 \, mm^4$ and $E = 2 \times 10^5 \, N/mm^2$ find

(i) Slope at free end

(ii) Deflection at free end

Given, Length, $l = 3m = 3000 \, mm$

point load, $W = 50kN = 50000 \, N$

Distance of load from fixed end,

$a = 2m = 2000 \, mm$

$I = 10^8 \, mm^4$

$E = 2 \times 10^5 \, N/mm^2$

(i) Slope at free end,

$$\theta_B = \theta_C = \frac{Wa^2}{2EI} = \frac{50000 \times \cancel{2000}^2 \times \cancel{4} \times 10^6}{\cancel{2} \times \cancel{2} \times 10^5 \times 10^8}$$

$$= 0.005 \text{ rad.}$$

(ii) Deflection at free end,

$$y = \frac{Wa^3}{3EI} + \frac{Wa^2}{2EI}(l-a)$$

$$= \frac{50000 \times 2^3 \times 10^9}{3 \times 2 \times 10^5 \times 10^8} + \frac{50000 \times 2^2 \times 10^6}{3 \times 2 \times 10^5 \times 10^8} [3000 - 2000]$$

$$= 11.67 \text{ mm}$$

UNIT-4 Torsion in shaft

Shaft :

- Shaft is define as rotating machine element usually circular (Solid or hollow) cross-section, which is used to transmit power from one part to another part.
- Shaft transmit power from machine that produces power to machine that absorbs power.
- Shaft is important element of machines.
- Shaft are made up of generally mild steel.

Function of shaft :

- Shaft supports rotating parts like gears and pulley.
- Shaft transmit power from one parts to another part.

Calculation of polor moment of inertia;

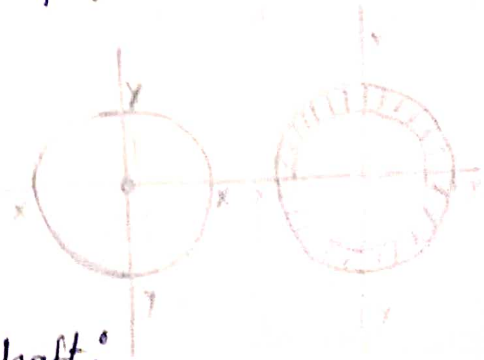
Polar Moment of inertia:

The measure of an objects ability to resist torsion in shaft is called polar moment of inertia.

According to perpendicular axis theorem,

$$I_{zz} = I_{xx} + I_{yy} = J = \text{polar moment of inertia.}$$

Polar moment of inertia for solid shaft;



$$I_{xx} = \frac{\pi d^4}{64}$$

$$I_{yy} = \frac{\pi d^4}{64}$$

$$I_{zz} = I_{xx} + I_{yy} = \frac{\pi d^4}{32}$$

Polar moment of inertia for hollow shaft;

$$I_{xx} = \frac{\pi}{64}(D^4 - d^4), \quad I_{zz} = I_{xx} + I_{yy} = \frac{\pi}{32}(D^4 - d^4).$$

$$I_{yy} = \frac{\pi}{64}(D^4 - d^4)$$

Assumption in simple torsion;

- The material of shaft is uniform throughout.
- The shaft circular remains circular after loading.
- Cross-section of shaft which is plane before twist remains plane after twist.
- The twist along the shaft is uniform throughout.
- Maximum shear stress induced in shaft due to application of torque doesnot exceeds its elastic value.
- All radii which are straight before twist remain straight after twist.

Torsion Equation :

The torsion equation is given by,

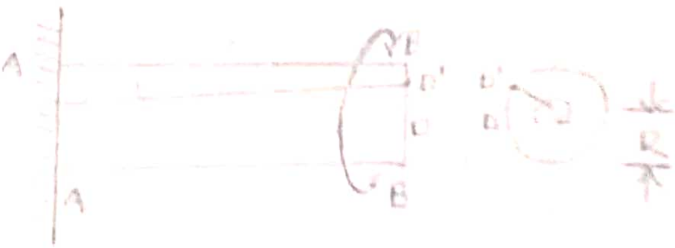
$$\boxed{\frac{T_R}{J} = \frac{\tau_{max}}{R} = \frac{G\theta}{L}}$$

Where, T_R = Resisting torque or resisting twisting couple.

Resisting torque develop due to two equal parallel and internal resisting shear force develop above and below the polar axis of cross-section of shaft.

$T_R \geq T$ - Safe condition

$T_R = T$ - Equilibrium Condition



L = Length of shaft

T = Maximum twisting couple

J = Polar moment of inertia

R = Radius of cross-section of shaft

G = Modulus of rigidity

θ = Angle of twist

τ_{max} = Maximum shear stress.

Analysis of Torsion Equation

$$\frac{T}{J} = \frac{\tau_{max}}{R} = \frac{G\theta}{L}$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$A \qquad B \qquad C$$

Case I ; $\frac{T}{J} = \frac{\tau_{max}}{R}$

$$\tau_{max} = \frac{TR}{J}$$

$$\boxed{\tau_{max} = \frac{T}{J/R} = \frac{T}{Z_P}} \quad -①$$

Where, Z_P = Polar section modlus of cross-section of shaft.

$$\boxed{Z_P = \frac{J}{R}}$$

Equation ① is used to determine τ_{max} develop on cross-section of shaft when twisting moment (T) is given,

From equation ① For given 'T',

$$\boxed{\tau_{max} \propto \frac{1}{Z_P}}$$

$Z_P\uparrow - \tau_{max}\downarrow \rightarrow$ changes in failure$\downarrow$

Case II ; $\frac{\tau_{max}}{R} = \frac{G\theta}{L}$

$$\boxed{\tau_{max} = \frac{G\theta R}{L}} \quad -②$$

equation ② is used to determine τ_{max} develop on cross-section of shaft when twisting angle is given,

Case III ; $\frac{T}{J} = \frac{G\theta}{L}$

$$\boxed{\theta = \frac{TL}{GJ}} \quad -③$$

equation ③ is used to determine maximum angle of twist of a cross-section of shaft when twisting moment is known.

GJ - Torsional rigidity of cross-section of shaft

$\theta = TL/GJ$, For given T and L

$\theta \propto \frac{1}{GJ}$; $GJ\uparrow \rightarrow \theta\downarrow \rightarrow \phi\downarrow \rightarrow \tau\downarrow$ - change in failure$\downarrow \rightarrow Z_P\uparrow$

Assumption in simple torsion;

- The material of shaft is uniform throughout.
- The shaft circular remains circular after loading.
- Cross-section of shaft which is plane before twist remain plane after twist.
- The twist along the shaft is uniform throughout.
- Maximum shear stress induced in shaft due to application of torque doesnot exceeds its elastic value.
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Torsion Equation:

The torsion equation is given by,

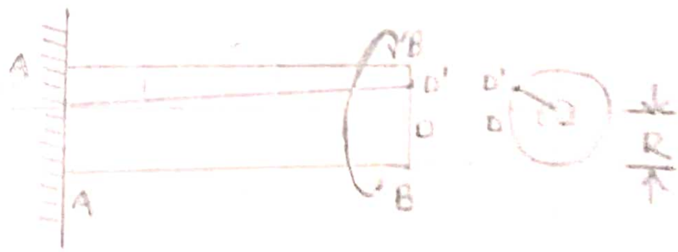
$$\boxed{\frac{T_R}{J} = \frac{\tau_{max}}{R} = \frac{G\theta}{L}}$$

Where, T_R = Resisting torque or resisting twisting couple.

Resisting torque develop due to two equal parallel and internal resisting shear force develop above and below the polar axis of cross-section of shaft,

$T_R \geq T$ - Safe condition

$T_R = T$ - Equilibrium Condition



L = Length of shaft

T = Maximum twisting couple

J = Polar moment of inertia

R = Radius of cross-section of shaft

G = Modlus of rigidity

θ = Angle of twist

τ_{max} = Maximum shear stress.

Analysis of Torsion Equation

$$\frac{T}{J} = \frac{\tau_{max}}{R} = \frac{G\theta}{L}$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$A \qquad B \qquad C$$

Case I ; $\frac{T}{J} = \frac{\tau_{max}}{R}$

$$\tau_{max} = \frac{TR}{J}$$

$$\boxed{\tau_{max} = \frac{T}{J/R} = \frac{T}{Z_P}} \quad -①$$

Where, Z_P = Polar section modlus of cross-section of shaft.

$$\boxed{Z_P = \frac{J}{R}}$$

Equation ① is used to determine τ_{max} develop on cross-section of shaft when twisting moment (T) is given,

From equation ① For given 'T',

$$\boxed{\tau_{max} \propto \frac{1}{Z_P}}$$

$Z_P\uparrow - \tau_{max}\downarrow \rightarrow$ changes in failure$\downarrow$

Case II ; $\frac{\tau_{max}}{R} = \frac{G\theta}{L}$

$$\boxed{\tau_{max} = \frac{G\theta R}{L}} \quad -②$$

equation ② is used to determine τ_{max} develop on cross-section of shaft when twisting angle is given,

Case III ; $\frac{T}{J} = \frac{G\theta}{L}$

$$\boxed{\theta = \frac{TL}{GJ}} \quad -③$$

equation ③ is used to determine maximum angle of twist of a cross-section of shaft when twisting moment is known.

GJ - Torsional rigidity of cross-section of shaft

$\theta = TL/GJ$, For given T and L

$\theta \propto \frac{1}{GJ}$; $GJ\uparrow \rightarrow \theta\downarrow \rightarrow \phi\downarrow \rightarrow \tau\downarrow$ - change in failure$\downarrow \rightarrow Z_P\uparrow$

Qn. What should to be length of a 5mm diameter alum? wire so that it can be twist through one complete revolution without exceeding a shearing stress of $42\ MN/m^2$. Take $G = 27\ GN/m^2$.

Qn. A solid steel shaft has to transmit 75 kW at 200rpm. Taking allowable shear stress as $75\ MN/m^2$. Find the suitable diameter for a shaft. If maximum torque transmitted on each revolution exceed the mean 80%.

Qn. A hollow shaft of diameter ratio 3/8 is required to transmit 600kW at 110 rpm, the max^m torque being 20% greater than the mean. The shear stress is not to exceeds $63\ MN/m^2$ and twist in a length of 8m not exceeds 1·4 degree. Calculate the maximum external diameter satisfying these condition. Take $G = 84\ GN/m^2$

[SOM - REC - 40]

Compresion of solid and hollow shaft.

1. Compresion by Strength;

Let us assume that both shaft have some length material, some weight and some shear stress.

Let D_S = Diameter of solid shaft,

d_H = internal diameter of hollow shaft,

D_H = External diameter of hollow shaft,

T_S = Torque transmitted by solid shaft,

T_H = Torque transmitted by hollow shaft.

$$\frac{d_H}{D_H} = k \text{ - assume}$$

Strength of hollow shaft, $T_H = \frac{\pi}{16}\left[\frac{D_H^4 - d_H^4}{D_H}\right]\tau$

$$T_H = \frac{\pi}{16}\frac{D_H^4\left(1 - \left(\frac{d_H}{D_H}\right)^4\right)}{D_H}\tau$$

$$\boxed{T_H = \frac{\pi}{16} D_H^3 (1 - k^4)\tau}$$

Strength of solid shaft;

$$T_S = \frac{\pi}{16} D_S^3 \tau$$

$$\Rightarrow \because \frac{T_H}{T_S} = \frac{\cancel{\pi/16}\, D_H^3 [1-k^4] \cancel{\tau}}{\cancel{\pi/16}\, D_S^3 \cancel{\tau}} = \frac{D_H^3 (1-k^4)}{D_S^3} \quad -(I)$$

As given, weight are same

$$W_S = W_H$$

$$A_S = A_H$$

$$\frac{\cancel{\pi}}{4} D_S^2 = \frac{\cancel{\pi}}{4} [D_H^2 - d_H^2]$$

$$D_S^2 = D_H^2 \left[1 - \left(\frac{d_H}{D_H}\right)^2\right]$$

$$D_S^2 = D_H^2 [1 - k^2]$$

$$\frac{D_S^2}{D_H^2} = 1 - k^2$$

$$\frac{D_H^2}{D_S^2} = \frac{1}{1-k^2}$$

$$\frac{D_H}{D_S} = \frac{1}{\sqrt{1-k^2}}$$

$$\left(\frac{D_H}{D_S}\right)^3 = \left(\frac{1}{1-k^2}\right)^{3/2} \quad -(II)$$

putting (II) in equation (I)

$$\frac{T_H}{T_S} = \frac{1}{(1-k^2)^{3/2}} \times (1-k^4)$$

$$\frac{T_H}{T_S} = \frac{\cancel{(1-k^2)}(1+k^2)}{\cancel{(1-k^2)}\sqrt{1-k^2}} \Rightarrow \boxed{\frac{T_H}{T_S} = \frac{1+k^2}{\sqrt{1-k^2}}}$$

This shows that, ~~torque~~ torque transmitted by hollow shaft is greater than solid shaft.

$\therefore$ Hollow shaft is stronger than solid shaft.

2. Compression by weight

Let us assume that both shaft have same length and material.

$$\frac{\text{Weight of hollow shaft}}{\text{Weight of solid shaft}}, \quad \frac{W_H}{W_S} = \frac{A_H}{A_S}$$

$$\frac{W_H}{W_S} = \frac{\cancel{\pi/4}(D_H^2 - d_H^2)}{\cancel{\pi/4} D_S^2} = \frac{D_H^2}{D_S^2}(1-k^2).$$

Torque applied in both shaft is same.

$$T_S = T_H$$

$$\frac{\cancel{\pi}}{\cancel{16}} D_S^3 \cancel{f} = \frac{\cancel{\pi}}{\cancel{16}} \frac{D_H^4 - d_H^4}{D_H} \cancel{f}$$

$$D_S^3 = \frac{\cancel{D_H}}{\cancel{D_H}}\left[D_H^3(1-k^4)\right]$$

$$\frac{D_H^3}{D_S^3} = \frac{1}{1-k^4}$$

$$\frac{D_H}{D_S} = \left(\frac{1}{1-k^4}\right)^{1/3}$$

$$\frac{D_H^2}{D_S^2} = \left(\frac{1}{1-k^4}\right)^{2/3}$$

$$\Rightarrow \quad \frac{W_H}{W_S} = \frac{1}{(1-k^4)^{2/3}} \times (1-k^2)$$

$$\Rightarrow \quad \frac{W_H}{W_S} = \frac{1-k^2}{(1-k^4)^{2/3}}$$

This shows that for same, material length and torque. Weight of hollow shaft will less than solid shaft.

∴ Hollow shaft is economical as compare to solid shaft.

Proof of Torsion equation

Let, T = Maximum twisting couple

R = Radius of shaft

J = Polar moment of inertia

τ = Shear stress

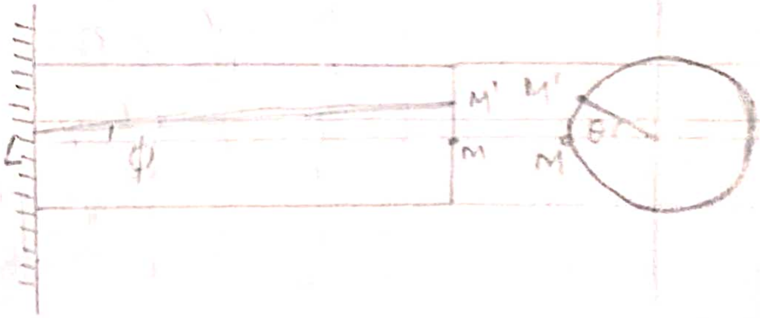
G = Modulus of rigidity

θ = Angle of twist

l = length of shaft

Consider a solid shaft fixed at one end and torque is applied at other end.

Let, LM is line drawn on the shaft it will distroted to LM' on application of the torque and therefore cross-section will twist through angle θ and surface by angle ϕ.



from $\Delta MLM'$

$$\tan\phi = \frac{MM'}{LM}$$

Since ϕ is very small

$$\tan\phi \cong \phi$$

$$\phi = \frac{MM'}{l} \quad -(1)$$

As we know from hook's law

$$\text{Modulus of rigidity} = \frac{\text{Shear stress}}{\text{Shear strain}}$$

$$G = \frac{\tau}{\phi}$$

$$\phi = \frac{\tau}{G} \quad -(2)$$

$$\frac{MM'}{l} = \frac{\tau}{G} \Rightarrow \frac{R\theta}{l} = \frac{\tau}{G}$$

$$\boxed{\frac{\tau}{R} = \frac{G\theta}{l}} \quad -(A)$$

Now,

Consider an elementry ring of thickness dx at radius x and let shear stress at this section be τ_x

The turning force on elementry ring

$$dF = \tau_x \times 2\pi x \, dx$$

Turning moment due to turning force

$$dT = dF \times x$$

$$dT = \tau_x \times 2\pi x \, dx \times x$$

$$dT = \tau_x \times 2\pi x^2 \, dx$$

$\therefore$ Total turning moment, T

$$T = \int_0^R \tau_x \times 2\pi x^2 dx$$

$$= \int_0^R \frac{\tau x}{R} 2\pi x^2 dx \qquad \left[\frac{\tau}{R} = \frac{\tau_x}{x} \because \tau_x = \frac{\tau x}{R}\right]$$

$$= \frac{\tau}{R} 2\pi \int_0^R x^3 dx$$

$$= \frac{\tau}{R} 2\pi \left[\frac{x^4}{4}\right]_0^R$$

$$= \frac{\tau 2\pi}{R} \times \left(\frac{R^4}{4} - 0\right)$$

$$\Rightarrow \frac{\tau R^3 \pi}{2} = \frac{\tau \pi R^3}{2}$$

$$T = \frac{\tau}{R} \times \frac{\pi R^4}{2} \Rightarrow \frac{\tau}{R} \times \frac{\pi (D/2)^4}{2} = \frac{\tau}{R} \frac{\pi D^4}{32}$$

$$T = \frac{\tau}{R} \times J$$

$$\boxed{\frac{\tau}{R} = \frac{T}{J}} \quad - (B)$$

From A and B

$$\therefore \quad \boxed{\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{l}}$$

Spring :

- Spring are elastic member which distort under load and regain their original shape when load is removed.
- Spring are used railway carriagus, motor car, motor-cycle etc.
- Spring are made of high carbon steel.

Function of spring;

- To absorb shock or impact loading as in carriage spring.
- To store energy in clock spring.
- To measure force in spring balance.

Types of spring ;

1. Laminated or leaf spring
2. Helical spring

Helical spring :

Helical spring is a length of wire or bar wound in helix.

Types of Helical spring,

- Open coil helical spring
- Close coil helical spring.

Open coil helical spring;

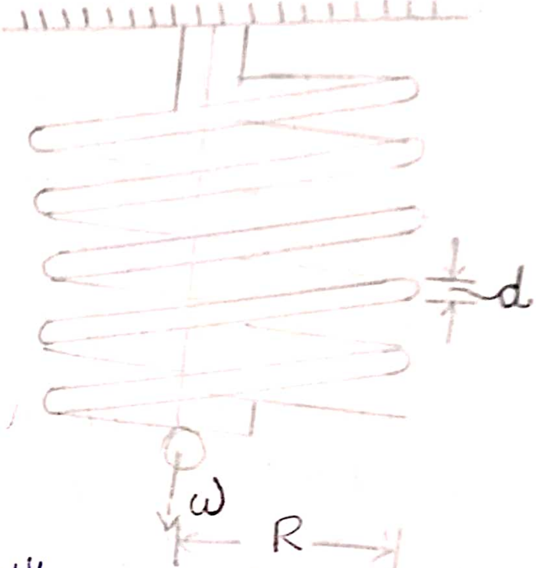
- These spring is also known by compression spring because these spring is used to resist compression.
- This spring has high pitch and not bound tightly.
- In this spring, there is a large gap between two consective coil.

Close coil Helical spring;

- This spring is designed to resist serectching and twisting.
- This spring is known by tension spring.
- In this spring, there is a hook at the ends.

Deflection of close coil helical spring

Let us consider a close coil helical spring loaded with axial load;



Let, R = Radius of coil

d = diameter of wire of coil

δ = Deflection of coil under load W.

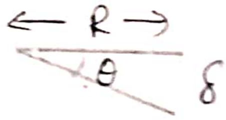
G = Modulus of rigidity

n = No. of coil or turn

l = length of wire $= 2\pi R n$

τ = Shear force

J = Polar moment of inertia $= \frac{\pi}{32} d^4$



Deflection;

$$\frac{T}{J} = \frac{G\theta}{l} = \frac{\tau}{r}$$

$$\Rightarrow \frac{T}{J} = \frac{\tau}{r}$$

$$\Rightarrow T = \frac{\pi}{16} d^3 \tau$$

$$\tau = \frac{16T}{\pi d^3} = \frac{16WR}{\pi d^3}$$

$$\frac{T}{J} = \frac{G\theta}{l}$$

$$\theta = \frac{Tl}{GJ}$$

$$\theta = \frac{WRl}{G \times \frac{\pi}{32} d^4} = \frac{64 W R^2 n}{C d^4}$$

$$\delta = R\theta \;; \quad \boxed{\delta = \frac{64 W R^3 n}{C d^4}}$$

Stifness of spring (k) :

$$K = \frac{\omega}{\delta}$$

$$\therefore \boxed{K = \frac{Cd^4}{64R^3n}}$$

Qn. A helical spring is made of 12 mm diameter steel wire wound on 120 mm diameter moudrel. If there are 10 active coils. What is spring constant ? Take $G = 82\ GN/m^2$. What force must be applied to spring to elongate it by 40mm ? [SOM-Rec-42]

Qn. A close coiled helical spring is to have a stifness of 900 N/m in compression with maximum load of 45 N and maximum shearing stress of $120\ N/mm^2$. The solid length of spring (ie coil touching) is 45 mm. Find, [SOM-REC-42]

a) Wire diameter

b) The mean coil radius

c) The no. of coil. Take $G = 0.4 \times 10^5 N/mm^2$

Given, ~~Stainless Steel~~

Stiffness of spring, $k = 900 N/m$

Max^M Load, $\omega = 45 N$

Max^M shear stress, $\tau = 120\ N/mm^2$

$G = 0.4 \times 10^5\ N/mm^2$

wire diffection,

$$\delta = \frac{64\ \omega R^3 n}{Cd^4}, \quad k = \frac{\omega}{\delta} = \frac{Cd^4}{64R^3n}$$

$$0.9 = \frac{0.4 \times 10^5 d^4}{64 \times R^3 \times n}$$

$$d^4 = \frac{0.9 \times 64 \times R^3 \times n}{0.4 \times 10^5}$$

Again, $\tau = \frac{16WR}{\pi d^3}$

$$120 = \frac{16 \times 45 \times R}{\pi d^3}$$

$$R = \frac{120 \pi d^3}{16 \times 45}$$

$$R = 0.52 d^3$$

Solid length of spring, $nd = 45$

$$n = \frac{45}{d}$$

$$\therefore d^4 = \left(\frac{0.9 \times 64}{0.4 \times 10^5}\right) (0.52\, d^3)^3 \times \frac{45}{d}$$

$$d^4 = \frac{0.9 \times 64}{0.4 \times 10^5} \times (0.52)^3 \times d^8 \times 45$$

$$d^4 = \frac{0.\cancel{4} \times 10^5}{0.9 \times \underset{18}{\cancel{64}} \times (0.52)^3 \times 45}$$

$$d = \sqrt[4]{\frac{10^5}{162 \times 0.14 \times 45}} = \sqrt[4]{\frac{10^5}{533.01}} = \sqrt[4]{187.61}$$

$$d = 3.24 \text{ mm.}$$

Mean coil radius

$$R = 0.52 \times d^3 = 0.52 \times (3.24)^3 = 17.68 \text{ mm}$$

No. of coil,

$$n = \frac{45}{d} = \frac{45}{3.24} = 13.88$$

Question above this one; –

Solⁿ, Diameter of wire, $d = 12$ mm

Diameter of mandrel, $D = 120$ mm

No. of active coil, $n = 10$, $G = 82\,GN/m^2$

Elongation of spring, $\delta = 40$ mm.

Let, spring constant be k,

$$\therefore k = \frac{Gd^4}{64R^3n}$$

$$k = \frac{82\times10^9 \times (0.012)^4}{64\times\left(\frac{0.12}{2}\right)^3\times10}$$

$$k = 12300 \text{ N/m}$$

Let force applied be ω,

$$k = \frac{\omega}{S}$$

$$\omega = kS$$

$$= 12300\times0.04$$

$$= 492 \text{ N}$$

Qn. What should to be length of a 5 mm diameter aluminium wire so that it can be twist through one complete revolution without exceeding a shearing stress of 42 MN/m². Take G = 27 GN/m².

Diameter of wire, $D = 5\text{mm}$

Angle of twist, $\theta = 2\pi$ rad

Shear stress, $\tau = 42 \text{ MN/m}^2 = 42 \text{ N/mm}^2$

$G = 27 \text{ GN/m}^2 = 27\times10^3 \text{ N/mm}^2$.

Let length of wire l,

As we know

$$\Rightarrow \frac{T}{J} = \frac{\tau}{R}$$

$$\Rightarrow \frac{T}{\frac{\pi}{32}D^4} = \frac{\tau}{D/2}$$

$$\Rightarrow T = \frac{\pi}{16}D^3\tau$$

$$T = \frac{\pi}{16}\times5^3\times42 = 1030.8 \text{ Nmm}$$

Again, $\frac{T}{J} = \frac{G\theta}{l}$

$$\frac{1030.8}{\frac{\pi}{32}5^4} = \frac{27\times10^3\times2\pi}{l}\text{; } l = 10098.32 \text{ mm}$$

$$= 10.09 \text{ m}$$

Qn. A solid steel shaft has to transmit 75 kW at 200 rpm. Taking allowable shear stress as 75 MN/m². Find suitable diameter for a shaft, if maximum torque transmitted on each revolution exceeds the mean 30%.

Power $P = 75$ kW

Speed, $N = 200$ rpm

Shear stress $\tau = 75\,MN/m^2$

$T_{max} = 1.3\,T_{mean}$

$\because \quad P = \dfrac{2\pi N T}{60}$

$75 \times 1000 = \dfrac{2\pi \times 200 \times T}{60}$

$T = \dfrac{75 \times 1000 \times 60}{2\pi \times 200} = 3581$ Nm

$T = T_{mean} = 3581$ Nm

$T_{max} = 1.3\,T_{mean}$

$= 1.3 \times 3581$

$= 4656.3$ Nm

$\because \quad T_{max} = \dfrac{\pi}{16} d^3 \tau$

$4656.3 = \dfrac{\pi}{16} d^3 \times 75 \times 10^6$

$d^3 = \dfrac{16 \times 4656.3}{\pi \times 75 \times 10^6}$, $d = 0.0697$ m $= 69.7$ mm

Qn. A hollow shaft of diameter ratio 3/8 is required to transmit 600 kW at 110 rpm, the maximum torque being 20% greater than the mean. The shear stress is not to exceeds 63 MN/m² and twist in a length of 3m not to exceeds 1.4 degree. Calculate the maximum external diameter satisfying these condition. Take G = 84 GN/m²

Let,

d be internal and D be external diameter of hollow shaf

$$\frac{d}{D} = \frac{3}{8}$$

$$\therefore d = 0.375D$$

$P = 600\,kW$

$N = 110\ rpm$

$T_{max} = 1.2\ T_{mean}$

$$P = \frac{2\pi N T}{60}$$

$$600 \times 10^3 = \frac{2\pi \times 110 \times T}{60}$$

$$T = \frac{600 \times 10^3 \times 60}{2\pi \times 110}$$

$T = T_{mean} = 52087\ Nm$

$T_{max} = 1.2\ T_{mean}$

$= 1.2 \times 52087$

$= 62504\ Nm$

Case -I, When shear stress not exceeds $63\,MN/m^2$

$$T = \frac{\pi}{16} \frac{(D^4 - d^4)}{D} \times \tau$$

$$\frac{62504 \times 16}{63 \times 10^6 \times \pi} = \frac{D^4 - (0.375)^4 D^4}{D} = D^3(1 - 0.375^4)$$

$$D^3 = \frac{62504 \times 16}{0.9802 \times 63 \times 10^6 \times \pi}$$

$D = 0.1727\ m = 172.7\ mm$

Case II, When angle of twist not to exceed $1.4°$

$$1.4\ \text{degree} = 1.4 \times \frac{\pi}{180}$$

$\therefore \frac{T}{J} = \frac{G\theta}{l}$

$T = \frac{G\theta}{l} \times J$

$62504 = \frac{84 \times 10^9 \times 1.4 \times \pi}{180 \times 3} \times \frac{\pi}{32} [D^4 - d^4]$

$62504 = \frac{84 \times 10^9 \times 1.4 \times \pi \times \pi}{180 \times 3 \times 32} \times D^4 [1 - 0.375]$

$D = 0.1755 m = 175.5 mm.$

$\therefore$ Extenal diameter maximum value,

$\underline{\underline{D = 175.5 \ mm}}$

| UNIT5 - Thin Cylindrical Shell

Pressure Vessel : The close cylindrical or spherical container design to store fluids at a pressure substained differed from ambient pressure is called pressure vessel.

Pressure Vessel

Thin Pressure Vessel

$\frac{d}{t} \geq 20$

Thick Pressure Vessel

$\frac{d}{t} < 20$

Shape of shell

(i) Cylindrical (ii) Spherical

Vessel of cylindrical and spherical form are used for storing fluid under pressure.

Example; Steam boiler, air compresser etc

Thin cylindrical shell :

- If the thickness wall of cylindrical vessel less than $\frac{1}{15}$ to $\frac{1}{20}$ of its internal diameter. This cylindrical vessel is called thin cylindrical shell.
- Boiler, Steam Pipes etc are considered as thin cylinder
- In thin cylinder, stress may be assumed uniformly distributed over the wall thichness.

Thin cylinder subjected to internal pressure;

Let,

the thin cylinde in which fluid is stored.

d = Internal dia of cylinder

t = thickness of wall

p = Internal pressure of pipe

l = length of thin cyinder

Due to intenal pressure P, the cylinder may blast or
by splitting up in any one of two ways.

a). The forces due to pressure of fluid acting vertically upward and downward on thin cylinder tends to brust the cylinder.

b). The forces, due to pressure of the fluid acting at the end of the thin cylinder tends to brust the thin cylinder.

Stress in thin cylinder pressure vessel subjected to internal pressure;

- When thin cylinder vessel is subjected to internal pressure, stress in the wall of cylinder is setup.
- The stresses set up on the wall of cylinder is on the cross-section along the axis and on this cross-section perpendicular to axis.
- This means that there will be two stresses will set up.
- Both the stresses will be tensile.
- Sresses are

a. Circular / circumferential / hoop stress

b. Longitudnal stress

Circumferential (Hoop) Stresses:

- The stress acting along the circumference of cylinder is called circumferential stress.
- It is denoted by σ_c.
- Consider thin pressure vessel subjected to internal pressure. The circumferential stress will be set up in the material of cylinder, if busting of cylinder take place.
- Let, d = Internal dia. of cylinder

  t = thickness of cylinder

  p = internal pressure of cylinder

  σ_c = Hoop stress

Busting force = Resisting force

$$p \times (d \times l) = (2l \times t)\,\sigma_c$$

$$\Rightarrow \boxed{\sigma_c = \frac{pd}{2t}}$$

Longitudnal Stresses:

- The stress acting along the length of the cylinder is called longitudnal stress. It is denoted by σ_l.
- Consider a cylindrical vessel subjected to internal pressure. The longitudnal stress will be set up in the material of cylinder if busting of cylinder take place.

Let, d = internal dia. of cylinder

p = internal pressure in cylinder

t = thickness of cylinder

σ_l = longitudnal stress

Busting force = Resisting force

$$p \times \frac{\pi}{4} d^2 = \pi d t \sigma_l \quad , \quad \boxed{\sigma_l = \frac{Pd}{4t}}$$

Note : Max^m Shear force,

$$\tau_{max} = \frac{\sigma_c - \sigma_l}{2}$$

$$\therefore \boxed{\tau_{max} = \frac{Pd}{8t}}$$

Efficiency of a Joint :

- The cylindrical shell has two types of joint which are as follow
  - a. Longitudnal Joint
  - b. Circumferential Joint
- In case of Joint holes are made in material of shell.
- Due to holes, the area of ~~resisting~~ resisting decreases.
- Due to holes the area, of ~~resisting~~ stress will be increase.
- Let μ_l = Efficiency of longitunal Joint

  μ_c = Efficiency of Circumferential Joint

Circuferential stress when efficiency is given,

$$\boxed{\sigma_c = \frac{Pd}{2t\mu_l}}$$

Longitudnal stress when efficiency is given,

$$\boxed{\sigma_l = \frac{pd}{4t\mu_c}}$$

Effect of internal pressure on dimension of thin cylinder.

Let, p = internal pressure

l = Length of cylinder

d = diameter of cylinder

t = thickness of cylinder

E = Young Modulus

σ_c = Hoop stress = $pd/2t$

σ_l = longitudnal stress = $pd/4t$

δd = Change in diameter
δl = Chang in length
ε_l = longitudnal strain
ε_c = Hoop strain

Circumference Strain,

$$\varepsilon_c = \frac{\sigma_c}{E} - \frac{\mu \sigma_l}{E}$$

$$\frac{\delta d}{d} = \frac{pd}{2tE} - \frac{\mu pd}{4tE}$$

$$\boxed{\delta d = \frac{d^2 p}{2tE}\left[1 - \frac{\mu}{2}\right]}$$

Longitudinal strain,

$$\varepsilon_l = \frac{\sigma_l}{E} - \frac{\mu \sigma_c}{E}$$

$$\frac{\delta l}{l} = pd/4tE - \frac{\mu pd}{2tE}$$

$$\boxed{\Rightarrow \delta l = \frac{lpd}{2tE}\left(\frac{1}{2} - \mu\right)}$$

Qn. A boiler shell is to be made of 15 mm thick plate having a limiting tensile stress of 120 N/mm² having a limitting tensile stress. If the efficiency of longitudnal and circumferential joint are 70% and 30% respectively. Determine,

a. The max^m permissible diameter of the shell for internal pressure 2 N/mm²

b. Permissible intensity of internal pressure when the shell dia is 1·5 m. [SoM - Rec - Old - 38]

Qn. A cylindrical pipe of diameter 1.5 m and thickn[illegible] is 1.5 cm is subjected to intenal fluid pressure of 1.2[illegible]. Find, (i) Longitudnal stress develop in pipe
(ii) Circumferential stress develop inpipe.

Qn. A cylinder of internal dia 2.5 m and thickness 5cm contain a gas. If tensile stress in the material is not to exceeds 80 N/mm^2. Determine internal pressure of gas.

Qn. A cylinder of inernal dia 0.5 m contain air at a pressure of 7 N/mm^2. If maximum permisible stress in the material is 80 N/mm^2. Find thickness of cylinder.

Qn. A thin cylinder of internal diameter 1.25 m contain a fluid at an internal pressure of 2 N/mm^2. Determine maximum thickness if
a. longitunal stress is not to exceeds 30 N/mm^2
b. Circumferential stress is not to exceeds 45 N/mm^2.

[SOM - Rec - Old - 37]

UNIT - 5

Qn. A boiler shell is to be made of 15 mm thick plate having a limiting tensile stress of $120 N/mm^2$. If the efficiencies of longitudnal and circumferential joint are 70% and 30%, respectively. Determine,

a. The maximum permissible diameter of the shell for internal pressure $2 N/mm^2$.

b. Permissible intensity of internal pressure when the shell diameter is 1.5 m.

Given, thickness, $t = 15$ mm

Eff. of Longitudnal stress $= \eta_l = 0.7$

Eff. of Circumferential stress $= \eta_c = 0.3$

Limiting inside stress $= 120 N/mm^2$

Internal pressure, $p = 2 N/mm^2$

a. Maximum permissible diameter of shell, for $p = 2 N/mm^2$

Maximum tensile stress, $\sigma_c = 120 N/mm^2$

$$\frac{pd}{2t\eta_l} = 120$$

$$\Rightarrow \frac{\cancel{2} \times d}{\cancel{2} \times 15 \times 0.7} = 120$$

$$\Rightarrow d = 120 \times 15 \times 0.7 = 1260 \text{ mm}$$

Maximum tensile stress, $\sigma_l = 120 N/mm^2$

$$\frac{pd}{4t\eta_c} = 120$$

$$\Rightarrow \frac{\cancel{2} \times d}{\cancel{4}_2 \times 15 \times 0.3} = 120$$

$$\Rightarrow d = 120 \times 15 \times 2 \times 0.3 = 1080 \text{ mm}$$

b. Permissible internal pressure for diameter

Maximum tensile stress, $\sigma_c = 120$

$$\frac{pd}{2t\eta_c} = 120$$

$$\frac{P \times 1.5 \times 1000}{2 \times 15 \times 0.7} = 120$$

$$P = \frac{120 \times 2 \times 15 \times 0.7}{1.5 \times 1000} = \underline{\underline{1.68\ N/mm^2}}$$

Maximum tensile stress, $\sigma_L = 120$

$$\frac{pd}{4t\eta_L} = 120$$

$$\frac{p \times 1.5 \times 1000}{4 \times 15 \times 0.3} = 120$$

$$p = \frac{4 \times 15 \times 120 \times 0.3}{1.5 \times 1000} = \underline{\underline{1.44\ N/mm^2}}$$

Qn. A cylindrical pipe of diameter 1.5 m and thickness is 1.5 cm is subjected to internal fluid pressure of 1.2 N/mm². Find,

(i) Longitudnal stress develop in pipe.

(ii) Circumferential stress develop in pipe.

Given,

Internal dia, $d = 1.5\ m$, $p = 1.2\ N/mm^2$

thickness, $t = 1.5\ cm = 1.5 \times 10^{-2}\ m$

$$\therefore \frac{d}{t} = \frac{1.5}{1.5 \times 10^{-2}} = 100.$$

$$\frac{d}{t} \geq 20 \text{ - thin cylinder,}$$

There fore, given cylindrical pipe is thin.

1. Longitudnal stress,

$$\sigma_l = \frac{pd}{4t}$$

$$= \frac{\overset{0.3}{\cancel{1.2}} \times \cancel{1.5}}{\cancel{4} \times \cancel{1.5} \times 10^{-2}} = \underline{30 \ N/mm^2}$$

2. Circumferential stress,

$$\sigma_c = \frac{pd}{2t}$$

$$= \frac{\overset{0.6}{\cancel{1.2}} \times \cancel{1.5}}{\cancel{2} \times \cancel{1.5} \times 10^{-2}} = \underline{\underline{60 \ N/mm^2}}$$

Qn. A cylinder of internal diameter 2.5m thickness 5cm contain a gas. If tensile stress in the material is not to exceeds $80 N/mm^2$. Determine internal pressure of gas.

Internal dia. $d = 2.5 m$

Thickness, $t = 5 m$

$$\frac{d}{t} = \frac{2.5}{5} = 50 \geq 20.$$

$\therefore$ Cylinder is thin.

Maximum tensile stress = Circumferential stress

$$\therefore \sigma_c = 80 \ N/mm^2$$

$$\frac{pd}{2t} = 80$$

$$\frac{p \times 2.5}{2 \times 5 \times 10^{-2}} = 80$$

$$p = \frac{\overset{14}{\cancel{80}} \times 2 \times 5 \times 10^{0}}{2.5 \times 10^{2}} = 3.2 \ N/mm^2.$$

Qn A cylinder of internal dia 0.5 m contain air at a pressure of 7 N/mm². If maximum permisible stress in the material is 80N/mm², Find thickness of cylinder.

Internal diameter, $d = 0.5$ m

Internal pressure, $p = 7 N/mm^2$

Maximum permisible stress = Circumferential stress

$\sigma_c = 80 N/mm^2$

$$\Rightarrow \frac{pd}{2t} = 80$$

$$\Rightarrow \frac{7 \times 0.5}{2t} = 80$$

$$\Rightarrow t = \frac{7 \times 0.5}{2 \times 800} = \frac{35}{1600} = 0.0218 \text{ m} = \underline{2.1 \text{ mm}}$$

Qn. A thin cylinder of internal diameter 1.25 m contain a fluid at an internal pressure of 2N/mm². Determine maximum thickness if :

a. Longitudnal stress is not to exceeds 30N/mm².

b. Circumferential stress is not to exceeds 45N/mm².

Given, Internal diameter, $d = 1.25$ m

Internal pressure, $p = 2 N/mm^2$

Circumferential stress, $\sigma_c = 45 N/mm^2$

Longitudnal stress, $\sigma_l = 30 N/mm^2$

$\sigma_l = 30 N/mm^2$

$$\frac{pd}{4t} = 30, \quad \frac{2 \times 1.25}{4t} = 30, \quad t = 0.0208 \text{ m}$$

$$= 2.08 \text{ mm}$$

$\sigma_c = 45 N/mm^2$

$$\frac{pd}{2t} = 45, \quad \frac{2 \times 1.25}{2 \times t} = 45, \quad t = 0.0277 \text{ m} = 2.77 \text{ mm}$$